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A STUDY OF THE INTERACTION OF ELECTROMAGNETIC WAVES WITH THE PLASMA SURROUNDING A RE-ENTRY VEHICLE

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ABSTRACT

An analysis is presented of the interaction of an electromagnetic wave with a plasma. In the first part of the paper general equations are derived for the dielectric constant, propagation function, and conductivity of a plasma. The second part of the paper deals with the attenuation and reflection characteristics of a homogenous, semi-infinite, uniform plasma slab. The analysis includes the effect of a magnetic field on attenuation and reflection.

A STUDY OF THE

INTERACTION OF ELECTROMAGNETIC WAVES

WITH THE PLASMA SURROUNDING A REENTRY VEHICLE

PREFACE

A plasma is an ionized gas containing an equal number of positive and negative charges. The ionosphere is one example. Another example is the sheath surrounding a high velocity reentry vehicle. This plasma sheath occupies the volume enclosed by the shock wave and the vehicle surface. Being a ponderable medium, it presents an obstacle to the unhindered propagation of electromagnetic radiation. This obstacle can be opaque, translucent or transparent in nature depending on the plasma's intrinsic properties and on the frequency and intensity of the incident radiation. Understanding the nature of the plasma sheath is fundamental to the solving of two urgent problems in space exploration and weapons systems design; ie., specifically:

1. The limiting or eliminating of the "reentry blackout";
2. The reduction of effective radar cross section.

Much theoretical and experimental effort has been devoted to these ends, but at present, it is difficult to consolidate the literature into a single comprehensive analysis containing both the theoretical background and useful programmable equations. For this purpose, the following report is proffered.

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PART I - INTRODUCTION

A. General Wave Equation

Maxwell's equations for a plasma are:

$$\text{curl } \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \qquad \text{eq. 1a}$$

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{eq. 1b}$$

$$\text{div } \vec{D} = \rho \qquad \text{eq. 1c}$$

$$\text{div } \vec{B} = 0 \qquad \text{eq. 1d}$$

where

$\vec{J}$ = current density = $\frac{\text{coulomb}}{\text{meter}^2\text{seconds}}$

$\vec{E}$ = electric field strength = $\frac{\text{newtons}}{\text{coulomb}}$

$\vec{D}$ = dielectric displacement = $\frac{\text{coulombs}}{\text{meter}^2}$

$\vec{B}$ = magnetic induction = $\frac{\text{newton seconds}}{\text{coulomb meter}}$

$\vec{H}$ = magnetic excitation = $\frac{\text{coulombs}}{\text{meter seconds}}$

ρ = charge density = $\frac{\text{coulombs}}{\text{meter}^3}$

where the MKS rationalized system of units is employed.

We will investigate the propagation of electromagnetic waves by ascribing some conductivity, σ, to the plasma, where σ may be a function of ω, n, v, ν, with

n = electron density (M^{-3})

ω = signal frequency (radians/S)

v = electron velocity (M/S)

ν = electron collision frequency (collisions/sec) *

The variables, n, ν and v are not freely chooseable as is ω, but depend on the mechanical and thermodynamical state of the reentry vehicle and its plasma sheath. At any given point in time, we can say that, theoretically at least, n, ν and v are determined by the temperature, T, of the environment,

*This unit is considered equivalent to radians per second in all ensuing equations.

by the space coordinates, x, y and z, by the composition of air and by the signal energy. These quantities are generally given indirectly through the altitude and velocity of the vehicle itself.

Let us initially make the following assumptions for a preliminary solution of eqs. 1;

Assume:

1. that the dielectric constant and magnetic permeability of the plasma are the same as for free space;
2. a linear relationship between the field quantities; i.e., from 1 and 2,

$$\vec{D} = \varepsilon_0 \vec{E} \qquad \text{eq. 2a}$$

$$\vec{B} = \mu_0 \vec{H} \qquad \text{eq. 2b}$$

3. the validity of Ohm's law (in the absence of a magnetic field); i.e.,

$$\vec{J} = \sigma \vec{E} \qquad \text{eq. 3}$$

4. that the effect of the probing wave energy can be neglected;
5. that the earth's magnetic field can be neglected.

The ranges of validity of 4 and 5 are discussed in the appendix. Having accepted 1-5, we can now rewrite eq. 1a as:

$$\text{curl } \vec{H} = \left[\varepsilon_0 \frac{\partial}{\partial t} + \sigma \right] \vec{E} \qquad \text{eq. 4}$$

But

$$\begin{aligned} \text{curl curl } \vec{E} &= - \text{curl} \left[\frac{\partial}{\partial t} (\mu_0 \vec{H}) \right] \\ &= - \mu_0 \frac{\partial}{\partial t} (\text{curl } \vec{H}) \end{aligned} \qquad \text{eq. 5}$$

Substituting eq. 4 into eq. 5 and employing the vector identity

$$\text{curl curl } \vec{a} = \nabla \times (\nabla \times \vec{a}) = \text{grad div } \vec{a} - \nabla^2 \vec{a} \qquad \text{eq. 6}$$

yields

$$-\mu_0 \frac{\partial}{\partial t} \left[\varepsilon_0 \frac{\partial}{\partial t} + \sigma \right] \vec{E} = \text{grad div } \vec{E} - \nabla^2 \vec{E} \qquad \text{eq. 7}$$

Now if we assume that the plasma is a neutral collection of electrons and ions, i.e., from eq. 1c and eq. 2a,

6. For a plasma, $\rho = 0$ or div E = 0,

then we have the following wave equation for the plasma:

$$\boxed{\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} + \mu_0 \sigma \frac{\partial \vec{E}}{\partial t} = \nabla^2 \vec{E}} \qquad \text{eq. 8}$$

If the properties of the plasma are uniform in the plane perpendicular to the direction of propagation, z, we can find a solution of eq. 8 where Ex and Ey are uniform in this plane and depend only on z and t directly and σ indirectly.

* * *

B. Generalized Dielectric Constant, ε

We are interested primarily in the spatial variation of the electric field. We can, therefore, choose the sinusoidal time variation and represent the field by

$$\vec{E} = \vec{E}\,(x,y,z)\, e^{j\omega t}, \qquad \text{eq. 9}$$

Substituting eq. 9 into the wave equation 8 yields

$$\nabla^2 \vec{E} + (\mu_0 \varepsilon_0 \omega^2 - j\omega\mu_0\sigma)\vec{E} = 0 \qquad \text{eq. 10}$$

But

$$\mu_0 \varepsilon_0 = \frac{1}{c^2} \qquad \text{eq. 11}$$

Hence,

$$\nabla^2 \vec{E} + \left(\frac{\omega^2}{c^2} - j\,\frac{\omega\sigma}{\varepsilon_0 c^2}\right)\vec{E} = 0 \qquad \text{eq. 12}$$

But

$$\frac{\omega}{c} = k_0 = \frac{2\pi}{\lambda} \qquad \text{eq. 13}$$

and we have finally

$$\nabla^2 \vec{E} + k_o^2 \left(1 - j\frac{\sigma}{\omega\varepsilon_o}\right)\vec{E} = 0 \qquad \text{eq. 14}$$

The free space analog to eq. 14 is the wave equation

$$\nabla^2 \vec{E} + k_o^2 \vec{E} = 0 \qquad \text{eq. 15}$$

We can therefore define a wave number for the plasma as

$$k = k_o \sqrt{1 - j\frac{\sigma}{\omega\varepsilon_o}} = \frac{\omega}{V} \qquad \text{eq. 16}$$

The index of refraction of the plasma is then given by

$$N = \frac{k}{k_o} = \frac{c}{V} = \sqrt{1 - j\frac{\sigma}{\omega\varepsilon_o}} \qquad \text{eq. 17}$$

Now the first of Maxwell's equations, eq. 1a, can be written

$$\text{curl } \vec{H} = (\varepsilon_o j\omega + \sigma)\vec{E} = j\omega\left(\varepsilon_o + \frac{\sigma}{j\omega}\right)\vec{E} \qquad \text{eq. 18}$$

if both E and H have time dependencies of $e^{j\omega t}$. Again, the free space analog of eq. 18 is

$$\text{curl } \vec{H} = j\omega\varepsilon_o\vec{E} \qquad \text{eq. 19}$$

Proceeding as before, we can define a generalized dielectric constant for the plasma as

$$\varepsilon_c = \varepsilon_o - j\frac{\sigma}{\omega} \qquad \text{eq. 20}$$

and a relative complex dielectric constant by

$$\boxed{\varepsilon = \frac{\varepsilon_c}{\varepsilon_o} = 1 - j\frac{\sigma}{\omega\varepsilon_o}} \qquad \text{eq. 21}$$

It follows then that

$$\varepsilon = N^2 = \frac{k^2}{k_o^2} \qquad \text{eq. 22}$$

and the wave equation becomes

$$\boxed{\nabla^2 \vec{E} + k_o^2 \varepsilon \vec{E} = 0} \qquad \text{eq. 23}$$

It must be remembered that dielectric "constant" is in general a misnomer. Dielectric "function" is much more appropriate, since the conductivity, σ, is usually spatially dependent.

* * *

C. The Propagation Function, γ

Let the solution of eq. 23 represent a plane wave traveling in the z direction and polarized in the xy plane; i.e.,

$$E_{x,y} = E_0 e^{-\gamma z}, \quad E_z = 0 \qquad \text{eq. 24}$$

Substituting eq. 24 into eq. 23 yields

$$\gamma^2 = -k_0^2 \varepsilon \qquad \text{eq. 25}$$

$$\boxed{\gamma = j k_0 \sqrt{\varepsilon} = \alpha + j\beta} \qquad \text{eq. 26}$$

In terms of the conductivity, σ, we have

$$\gamma = j k_0 \sqrt{1 - j \frac{\sigma}{\omega \varepsilon_0}} \qquad \text{eq. 27}$$

where γ is the propagation function, a complex number whose real part, α, represents the attenuation of the field amplitude and whose imaginary part, β, represents the change in phase in traversing the plasma; if we represent ε by

$$\varepsilon = \varepsilon_r + j \varepsilon_i \qquad \text{eq. 28}$$

where, for the complex conductivity

$$\sigma = \sigma_r + j \sigma_i \qquad \text{eq. 29}$$

we get

$$\varepsilon = \left(1 + \frac{\sigma_i}{\omega \varepsilon_0}\right) - j \frac{\sigma_r}{\omega \varepsilon_0} \qquad \text{eq. 30}$$

We can then express α and β as

$$\alpha = k_0 \sqrt{\frac{|\varepsilon| - \varepsilon_r}{2}} \quad \text{nepers/M} \qquad \text{eq. 31}$$

$$\beta = k_0 \sqrt{\frac{|\varepsilon| + \varepsilon_r}{2}} \quad \text{radians/M} \qquad \text{eq. 32}$$

or, in terms of σ_r and σ_i :

$$\alpha = k_0 \left[\frac{-\left(1 + \frac{\sigma_i}{\omega\varepsilon_0}\right) + \sqrt{\left(1+\frac{\sigma_i}{\omega\varepsilon_0}\right)^2 + \left(\frac{\sigma_r}{\omega\varepsilon_0}\right)^2}}{2} \right]^{\frac{1}{2}} \quad \text{nepers/M} \qquad \text{eq. 33}$$

$$\beta = k_0 \left[\frac{\left(1 + \frac{\sigma_i}{\omega\varepsilon_0}\right) + \sqrt{\left(1+\frac{\sigma_i}{\omega\varepsilon_0}\right)^2 + \left(\frac{\sigma_r}{\omega\varepsilon_0}\right)^2}}{2} \right]^{\frac{1}{2}} \quad \text{radians/M} \qquad \text{eq. 34}$$

If the conductivity, σ , is known, then γ is determined and its sign is chosen such as α produces an attenuation rather than an amplification. Similarly, for the field expression of eq. 24, σ is determined for a given signal frequency if γ at that frequency is known.

* * *

D. Boltzmann's Equation and the Collision Frequency, ν

The conductivity of the plasma is due to the motion of electrons and ions under the influence of the $\vec{E}$ field. The complex nature of the plasma makes an exact determination of σ impossible. A simplified model is generally employed, and the test of its effectiveness is the ultimate correlation with experimental data. Since the ions are much more massive than the electrons, they are usually considered immobile, and hence the conductivity depends chiefly on electronic motion. The kinetic equation which describes this motion is generally the Boltzmann equation, which has its origin in the kinetic theory of gases:[4]

$$\boxed{\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla f + \frac{1}{m}\left[e\vec{E} + e\vec{v}\times\vec{B}\right]\cdot\nabla_v f = \frac{\delta f}{\delta t}}$$ eq. 35

where $\vec{E}$ is the probing field, $\vec{B}$ is an external magnetic field, e, $\vec{v}$ and m are the electronic charge, velocity and mass respectively, $\frac{\delta f}{\delta t}$ is the time rate of change of f at a fixed point due to electron collisions, and $f = f(\vec{r},\vec{v},t)$ is the distribution function for the electrons, with $[f(\vec{r},\vec{v},t)\cdot d\vec{r}\, d\vec{v}]$ being the probable number of electrons, which at time, t, have position coordinates $\vec{r}$ between $\vec{r}$ and $\vec{r} + d\vec{r}$ and velocity coordinates, $\vec{v}$, between $\vec{v}$ and $\vec{v} + d\vec{v}$.

Let us now assume that $\vec{B} = 0$ and that $\vec{E}$ is small and introduces a small perturbation f_1 into the unperturbed distribution function f_o (i.e., approximation 4, page 4) then[1]

$$f = f_o + f_1$$ eq. 36

and

$$\frac{\partial f_o}{\partial t} + \vec{v}\cdot\nabla f_o = \frac{\delta f_o}{\delta t}$$ eq. 37

Then substituting eq. 36 and eq. 37 into Boltzmann's eq. 35, neglecting products of the small quantities ($\vec{E}$ and f_1) and letting f_1 be independent of the space coordinates, yields

$$\frac{\partial f_1}{\partial t} + \frac{e\vec{E}}{m}\cdot\nabla_v f_o = \frac{\delta f_1}{\delta t}$$ eq. 38

$\frac{\delta f_1}{\delta t}$ can be represented by

$$\frac{\delta f_1}{\delta t} = \left.\frac{\delta f_1}{\delta t}\right|_a + \left.\frac{\delta f_1}{\delta t}\right|_i + \left.\frac{\delta f_1}{\delta t}\right|_e$$ eq. 39

where the change in f_1 is composed of contributions due to electron-atom, electron-ion and electron-electron collisions.

The general expression is

$$\left(\frac{\delta f_1}{\delta t}\right)_2 = \iiint (f_1' f_2' - f_1 f_2)\, g b \, db \, d\epsilon \, dv_2 \qquad \text{eq. 40}$$

representing the change in f_1 due to collisions of electrons (subscript 1) with other particles denoted by 2 (atoms, ions or electrons). Only binary encounters are considered, implying that the plasma is of relatively low density. The probable number of electrons in $d\vec{r}$ having velocities in the range $\vec{v}_1$, $d\vec{v}_1$, is $f_1 \, d\vec{v}_1 \, d\vec{r}$; likewise the probable number of particles of the second kind (a, i, e) in $d\vec{r}$ having velocities in the range $\vec{v}_2$, $d\vec{v}_2$, is $f_2 \, d\vec{v}_2 \, d\vec{r}$; g is the relative velocity of the two particles and is the same before and after collision; only its direction is changed. b is the impact parameter, i.e., the distance of closest approach of the two particles if they continued to move in straight lines with their initial velocities and were not acted upon by mutual forces; therefore, b is likewise the same before and after collision. ϵ specifies the orientation of the planes of the two paths.

The average number of collisions undergone by each particle per unit time is called the collision frequency. Chapman expresses this as[4]

$$C.F._{12} = \frac{N_{12}}{n_1} = \frac{2 n_1 n_2 s_{12}^2}{n_1} \left[\frac{2\pi k T m_0}{m_1 m_2}\right]^{\frac{1}{2}} \qquad \text{eq. 41}$$

$$C.F._{11} = \frac{N_{11}}{n_1} = \frac{4 n_1^2 s_1^2}{n_1} \left[\frac{\pi k T}{m_1}\right]^{\frac{1}{2}} \qquad \text{eq. 42}$$

The frequency for all collisions is $(N_{11} + N_{12} + \dots)/n_1$,

where n_i is the number density of the i'th particle,

$$s_{12} = 1/2\ (s_1 + s_2) \qquad \text{eq. 43}$$

and s_1 is the diameter of the i'th particle; also

$$m_0 = m_1 = m_2 \qquad \text{eq. 44}$$

A direct correlation between the collision frequency and $\frac{\delta f_1}{\delta t}$ is difficult to make, since the latter represents the net change in f_1 due to collisions, not the total number of collisions. At the present time, there are no reliable closed form expressions for eq. 40 which would readily lead to a solution of the differential equation 38. In fact, since coulomb forces predominate in a high temperature plasma, Boltzmann's equation itself may be inaccurate and multiple particle collisions must be taken into account. The lack of good experimental data to date necessitates approximations which in the light of further evidence may prove invalid.

With due consideration to the above limitations, a derivation of the plasma conductivity, σ, can be attempted.

* * *

E. Derivation of the Plasma Conductivity, σ

1. Without a magnetic field.

Golant[1] reports that, to the accuracy of the mass ratio of the electron and the heavy particle, $\frac{\delta f_1}{\delta t}$ can be represented by

$$\frac{\delta f_1}{\delta t} = -\nu f_1 = -(\nu_{ea} + \nu_{ei}) f_1 \qquad \text{eq. 45}$$

where

$$\nu_{ea} = n_a S_{ea} v; \quad \nu_{ei} = n_i S_{ei} v \qquad \text{eq. 46}$$

$$S = \int_{\Omega} c\,(1 - \cos\theta)\, d\Omega \qquad \text{eq. 47}$$

Elastic collisions are considered predominant, n_a and n_i are the concentrations of atoms and ions, c is the differential scattering cross section, θ is the scattering angle; the integration is taken over the entire scattering angle.

Therefore, eq. 38 becomes

$$\frac{\partial f_1}{\partial t} + \frac{eE}{m}\frac{\partial f_0}{\partial v} = -\nu f_1 + \frac{\delta f_1}{\delta t}\Big|_{ee} \qquad \text{eq. 48}$$

Neglecting electron-electron collisions (weakly ionized plasma) and defining an effective collision frequency, ν_e , averaged over the velocity distribution by

$$\nu_{ea}\bar{v} = \int_v \nu f_1 v\, d\vec{v} \qquad \text{eq. 49}$$

where

$$\bar{v} = \int v f_1\, d\vec{v} \qquad \text{eq. 50}$$

leads to the following equations from eq. 48:

$$\frac{\partial f_1}{\partial t} + \frac{eE}{m}\frac{\partial f_0}{\partial v} = -\nu f_1 \qquad \text{eq. 51}$$

Multiply by v and integrate over the entire velocity space

$$\int v \frac{\partial f_1}{\partial t}\, dv + \int v \frac{eE}{m}\frac{\partial f_0}{\partial v}\, dv = \int -\nu f_1 v\, dv \qquad \text{eq. 52}$$

$$\frac{d}{dt}\int v f_1\, dv + \frac{eE}{m}\left\{ \left[f_0 v\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f_0\, dv \right\} = -\int \nu f_1\, dv \qquad \text{eq. 53}$$

$$\frac{d\bar{v}}{dt} + \frac{eE}{m}(-1) = -\nu_e \bar{v} \qquad \text{eq. 54}$$

or

$$\boxed{m\frac{d\bar{v}}{dt} = e\bar{E}\, e^{j\omega t} - m\nu_e \bar{v}} \qquad \text{eq. 55}$$

for a time varying velocity independent field.

If ν is independent of v, then $\nu_e = \nu$, and the solution of eq. 55 is

$$\bar{v} = \frac{eEe^{j\omega t}}{m(\nu + j\omega)} \qquad \text{eq. 56}$$

Therefore, since

$$\vec{J} = ne\vec{v} = \sigma\vec{E} \qquad \text{eq. 57}$$

we have

$$\boxed{\begin{aligned} \sigma &= \frac{ne^2}{m} \cdot \frac{1}{\nu + j\omega} \\ \sigma_r &= \frac{ne^2}{m} \cdot \frac{\nu}{\omega^2 + \nu^2} \\ \sigma_i &= -\frac{ne^2}{m} \cdot \frac{\omega}{\omega^2 + \nu^2} \end{aligned}} \qquad \begin{aligned} &\text{eq. 58a} \\ &\text{eq. 58b} \\ &\text{eq. 58c} \end{aligned}$$

When $\omega \gg \nu$,

$$\sigma \cong j\sigma_i \cong -j\frac{ne^2}{\omega m} \qquad \text{eq. 59}$$

If ν is proportional to the velocity, i.e.,

$$\nu = av \qquad \text{eq. 60}$$

then

$$\sigma_i = -K_i \frac{ne^2}{m} \cdot \frac{\omega}{\omega^2 + \nu^2} \qquad \text{eq. 61a}$$

$$\sigma_r = K_r \frac{ne^2}{m} \cdot \frac{\nu}{\omega^2 + \nu^2} \qquad \text{eq. 61b}$$

for a Maxwellian distribution where the K's are given in Table 1.

	Highly Ionized Gas		Weakly Ionized Gas	
$\frac{\omega}{\nu}$	K_i	K_r	K_i	K_r
0.00	1.51	1.13	4.59	1.95
0.05	1.50	1.13	4.51	1.92
0.10	1.48	1.12	4.34	1.86
0.20	1.40	1.03	3.79	1.65
0.50	1.19	1.02	2.30	1.07
1.00	1.07	0.94	1.41	0.72
2.00	0.99	0.95	1.05	0.62
4.00	1.00	0.98	0.97	0.73
6.00	1.00	0.99	0.98	0.82
10.0	1.00	1.00	0.99	0.92
35.0	1.00	1.00	1.00	0.99
∞	1.00	1.00	1.00	1.00

TABLE 1

Fang[2] expressed the current density, $\vec{J}$, as

$$\vec{J} = \sigma \vec{E} = n e \cdot \frac{e \vec{E}_0}{m} \left(I_1 \cos \omega t + I_2 \sin \omega t \right) \qquad \text{eq. 62}$$

where

$$I_1 = \frac{8}{3\sqrt{\pi}} \int_0^{\infty} e^{-u^2} \cdot \frac{\nu u^4}{\omega^2 + \nu^2} \, du \qquad \text{eq. 63a}$$

$$I_2 = \frac{8}{3\sqrt{\pi}} \int_0^{\infty} e^{-u^2} \cdot \frac{\omega u^4}{\omega^2 + \nu^2} \, du \qquad \text{eq. 63b}$$

and v = velocity of electrons (M/s)

k = Boltzmann's constant = 1.38×10^{-23} joules/deg.mole

e = 1.6×10^{-19} coulombs

m = 9.1×10^{-31} kg

ν = collision frequency (collisions/sec) or radians/sec.

$v_0 = \left[\frac{2kT}{m} \right]^{\frac{1}{2}}$

$u = \frac{v}{v_0}$

Bachynski[3] writes the conductivity as

$$\sigma = -\frac{4\pi}{3}\frac{e^2}{m}\int_0^\infty \frac{v^3}{\nu + j\omega}\frac{\partial f_0}{\partial v}\,dv \qquad \text{eq. 64}$$

Using the "normalized" Maxwell distribution,

$$f_0 = n\left[\frac{m}{2\pi kT}\right]^{\frac{3}{2}} e^{-\frac{mv^2}{2kT}} \qquad \text{eq. 65}$$

eq. 64 reduces to

$$\sigma = \frac{ne^2}{m}\frac{8}{3\sqrt{\pi}}\int_0^\infty \frac{\nu - j\omega}{\nu^2+\omega^2}\, u^4 e^{-u^2}\,du \qquad \text{eq. 66}$$

Comparing eq. 66 with eq. 63, we see that

$$\sigma_r = \frac{ne^2}{m}\cdot I_1 \qquad \text{eq. 67a}$$

$$\sigma_i = -\frac{ne^2}{m}\cdot I_2 \qquad \text{eq. 67b}$$

A numerical table for the integrals I_1 and I_2 is given in Dingle et. al.[5]

If $\nu = \nu(u)$ is known, the above integrals can be solved. The exact form of ν, however, is generally not known. For air in the range 6000-12000°K, ν is approximately constant[3]. Then the above results agree with Golant.[1]

A recent analysis[6] allows for a non-Maxwellian velocity distribution. For our purposes, though, the expressions for σ given in eqs. 58 will suffice for this report; i.e., ν is considered constant and the effects of electron-electron collisions are ignored. In a later report consideration will be made of collision frequencies either proportional to velocity or empirically determined.

2. Uniform magnetic field

In the presence of a uniform magnetic field, the conductivity, σ, becomes

a tensor and Ohm's law assumes the form

$$\vec{J} = \|\sigma\| \vec{E} \qquad \text{eq. 68}$$

where

$$\|\sigma\| = \begin{Vmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{Vmatrix} \qquad \text{eq. 69}$$

If the magnetic field is directed along the z-axis and if the direction of propagation is unspecified, then[7]

$$\|\sigma\| = \begin{Vmatrix} b+c & j(b-c) & 0 \\ -j(b-c) & b+c & 0 \\ 0 & 0 & d \end{Vmatrix} \qquad \text{eq. 70}$$

where

$$d = \frac{ne^2}{m} \frac{1}{\langle \nu_g \rangle g_0 + j\omega h_0} \qquad \text{eq. 71a}$$

$$2c = \frac{ne^2}{m} \frac{1}{\langle \nu_g \rangle g_+ + j(\omega + \omega_B) h_+} \qquad \text{eq. 71b}$$

$$2b = \frac{ne^2}{m} \frac{1}{\langle \nu_g \rangle g_- + j(\omega - \omega_B) h_-} \qquad \text{eq. 71c}$$

and

$$\omega_B = \frac{eB}{m} = 1.759 \times 10^{11} B\left(\tfrac{\text{webers}}{M^2}\right) = 1.759 \times 10^{7} B(\text{gauss}) \text{ radians/s} \qquad \text{eq. 72}$$

$$\langle \nu_g \rangle = -\frac{4\pi}{3n} \int_0^\infty \frac{\partial f_0}{\partial v} \cdot v^3 \cdot \nu_g \, dv \qquad \text{eq. 73}$$

We are primarily concerned with the case of parallel propagation; i.e., a plane wave polarized in the xy plane and traveling along the z axis. Consider Fig. 1. The x-component of the field can be represented by the sum of two rotating vectors in the opposite direction. If the wave is receding, we designate the clockwise vector as right-handed (+) and the counter-clockwise

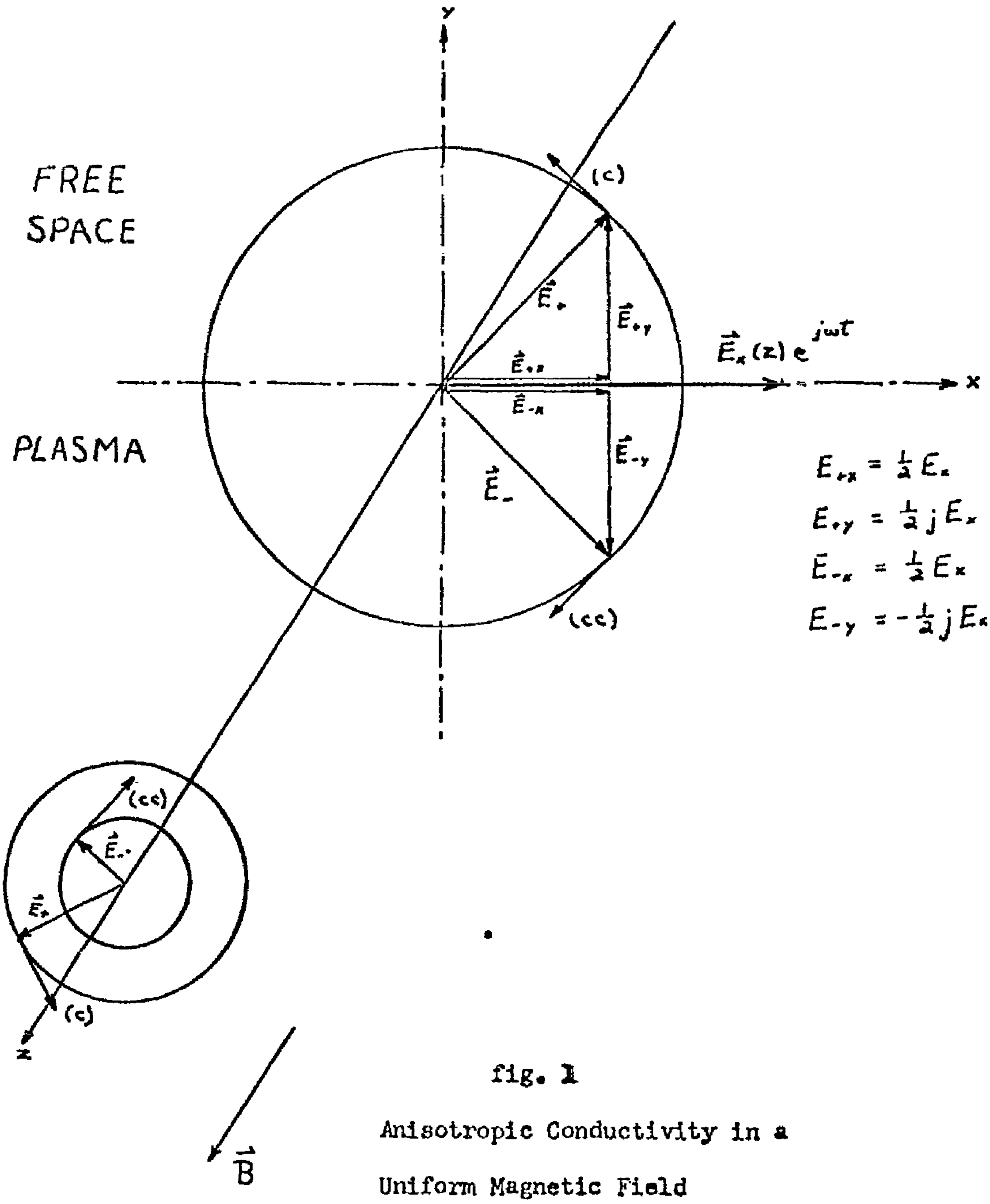

fig. 1

Anisotropic Conductivity in a Uniform Magnetic Field

vector as left handed (-). Ohm's law for the rotating vectors can then be written

$$\vec{J}_+ = \sigma_+ \vec{E}_+ \qquad \text{eq. 74a}$$

$$\vec{J}_- = \sigma_- \vec{E}_- \qquad \text{eq. 74b}$$

In the absence of a magnetic field, these vectors would rotate at the signal frequency, ω. The electrons, however, rotate also at the gyrofrequency, ω_B, due to the constant magnetic field. The conductivities then assume the new values

$$\sigma_+(\omega) = \sigma(\omega - \omega_B) \qquad \text{eq. 75a}$$

$$\sigma_-(\omega) = \sigma(\omega + \omega_B) \qquad \text{eq. 75b}$$

i.e., functions of both frequencies. (If the traveling field possessed a z-component, it would be unaffected by the magnetic field due to the vector product form of the force equation. Therefore, the conductivity along the z-axis remains as before.) It is clear that the right and left-handed conductivities are different. In fact, as we will see later, for certain field strengths, the right-handed mode is propagated through the plasma with less attenuation than the left handed. It should also be noted that σ_{ij} implies the conductivity along the i'th axis due to the field component directed along the j'th axis. A component analysis, remembering that a clockwise rotation is imparted to the electrons due to the magnetic field, leads to the following expression:

$$\|\sigma\| = \left\| \begin{matrix} \frac{1}{2}(\sigma_+ + \sigma_-) & -j\frac{1}{2}(\sigma_+ - \sigma_-) & 0 \\ \frac{1}{2}(\sigma_+ - \sigma_-) & \frac{1}{2}(\sigma_+ + \sigma_-) & 0 \\ 0 & 0 & \sigma \end{matrix} \right\| \qquad \text{eq. 76}$$

Substituting the conductivity expressions derived in the last section for constant collision frequency yields

$$\sigma_{\pm r} = \frac{ne^2}{m} \cdot \frac{\nu}{\nu^2 + (\omega \mp \omega_B)^2} \qquad \text{eq. 77a}$$

$$\sigma_{\mp i} = -\frac{ne^2}{m} \cdot \frac{(\omega \mp \omega_B)}{\nu^2 + (\omega \mp \omega_B)^2} \qquad \text{eq. 77b}$$

from eqs. 58.

* * * * x

Part II - The Homogeneous Semi-Infinite Uniform Plasma Slab

A. Attenuation Analysis

1. Without a magnetic field

The first and the simplest case we will investigate is the plasma described in fig. 2; i.e., a plane wave normally incident

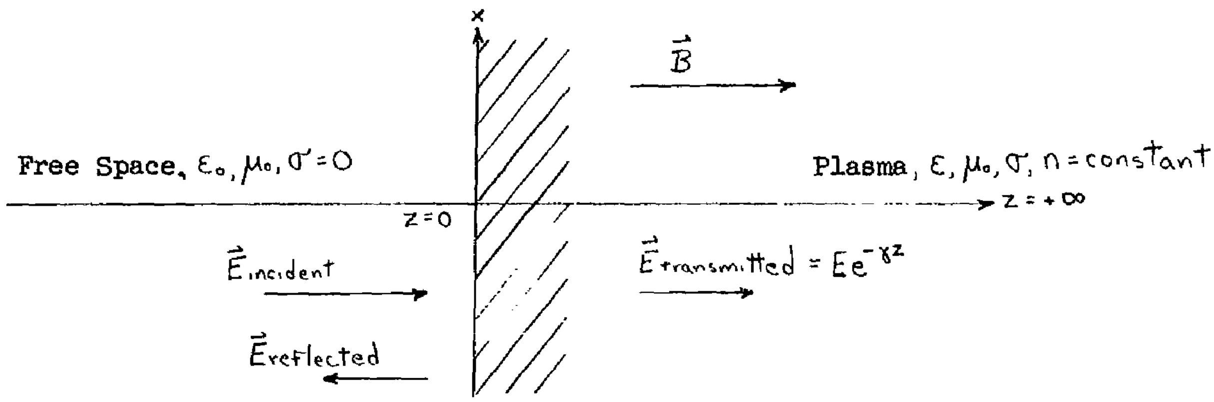

Fig. 2

on the plasma at z = 0. Within the plasma, the field is given by eg. 24 of Part I:

$$E = Ee^{-\gamma z} \qquad \text{eq. 24}$$

The attenuation is therefore,

$$\alpha = \frac{\omega}{c\sqrt{2}}\left[-\left(1+\frac{\sigma_i}{\omega\epsilon_0}\right)+\sqrt{\left(1+\frac{\sigma_i}{\omega\epsilon_0}\right)^2+\left(\frac{\sigma_r}{\omega\epsilon_0}\right)^2}\,\right]^{\frac{1}{2}} \text{ nepers/M} \qquad \text{eq. 33}$$

If we now substitute the conductivity expressions derived in Part I in the absence of a magnetic field, we get from eqs. 58 and 33,

$$\alpha = \frac{\omega}{c\sqrt{2}}\left[-1+\frac{ne^2}{\epsilon_0 m}\cdot\frac{1}{\nu^2+\omega^2} + \sqrt{1-2\,\frac{ne^2}{\epsilon_0 m}\cdot\frac{1}{\nu^2+\omega^2}+\left(\frac{ne^2}{\epsilon_0 m}\right)^2\frac{1}{\omega^2(\nu^2+\omega^2)}}\,\right]^{\frac{1}{2}} \qquad \text{eq. 78}$$

Let us designate the quantity

$$\omega_p = \left(\frac{ne^2}{\epsilon_0 m}\right)^{\frac{1}{2}} = 5.633 \times 10^4 \sqrt{n} \quad \text{radians/s} \qquad \text{eq. 79}$$

as the plasma frequency, where n is given in electrons/cm^3.*

In terms of ω_p we can now write the attenuation in its final form:

$$\alpha = 8.686 \frac{\omega}{c\sqrt{2}} \left[\frac{\omega_p^2}{\omega^2 + \nu^2} - 1 + \sqrt{1 + \frac{\omega_p^4 - 2\omega^2\omega_p^2}{\omega^2(\omega^2+\nu^2)}} \right]^{\frac{1}{2}} \text{db/cm} \qquad \text{eq. 80}$$

where c = 3×10^{10} cm/sec.

Eq. 80 has been programmed on the Philco 2000 digital computer at Chrysler Corporation's Missile Division.** We have considered the single frequency of 240 megacycles (1.507 radians/second) in this report, but the program is capable of computing any of the unknown quantities on the abscissa or ordinate.

Let us now consider the behavior of the attenuation in eq. 80 as the variables get very large or very small. Recall that

$$\sigma_r = \frac{ne^2}{m} \cdot \frac{\nu}{\omega^2 + \nu^2} \qquad \text{eq. 58b}$$

$$\sigma_i = -\frac{ne^2}{m} \cdot \frac{\omega}{\omega^2 + \nu^2} \qquad \text{eq. 58c}$$

$$\gamma = j\frac{\omega}{c}\sqrt{1 - j\left(\frac{\sigma_r}{\omega\epsilon_0} + j\frac{\sigma_i}{\omega\epsilon_0}\right)} = \alpha - j\beta \qquad \text{eq. 27}$$

Now if we let ω get large with respect to the other variables, we see that

$$\lim_{\omega\to\infty} \sigma_r = \lim_{\omega\to\infty} \sigma_i = 0 \qquad \text{eq. 81}$$

* As with ω_B, we choose to depart from MKS units purely from a practical standpoint.

**Program title: Minimum Attenuation, 9-16 - MA.

and consequently,

$$\lim_{\omega \to \infty} \gamma = j \frac{\omega}{c} = j\beta \qquad \text{eq. } 82$$

i.e., the propagation constant, γ , becomes pure imaginary.

Hence,

$$\lim_{\omega \to \infty} \alpha = 0 \qquad \text{eq. } 83$$

Therefore, an electromagnetic wave at a high enough frequency can pass through the plasma with virtually no attenuation, depending primarily on the electron density.

If the collision frequency is large compared to the signal frequency, both the real and imaginary parts of the conductivity vanish as in eq. 81, and γ becomes pure imaginary. Consequently,

$$\lim_{\nu \to \infty} \alpha = 0 \qquad \text{eq. } 84$$

This low attenuation at high collision frequencies occurs because the electrons, having a mean free path of small duration, do not have enough time to abstract energy from the incoming wave and convert it into incoherent radiation.[8]

If the collision frequency is very small compared to the signal frequency, we have

$$\lim_{\nu \to 0} \sigma = -j \frac{ne^2}{\omega m} \qquad \text{eq. } 59$$

Consequently,

$$\lim_{\nu \to 0} \gamma = j \frac{\omega}{c} \sqrt{1 - \frac{ne^2}{\omega m}} \qquad \text{eq. } 85$$

$$\lim_{\nu \to 0} \gamma = j \frac{\omega}{c} \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \qquad \text{eq. 86}$$

$$= \frac{1}{c} \sqrt{\omega_p^2 - \omega^2} \qquad \text{eq. 87}$$

If, in eq. 87, $\omega \geq \omega_p$, then γ is either pure imaginary or identically zero. In both cases, the attenuation is zero:

$$\lim_{\nu \to 0} \alpha = 0, \quad \text{if } \omega \geq \omega_p \qquad \text{eq. 88}$$

If, on the other hand, $\omega < \omega_p$, γ is everywhere real, and

$$\lim_{\nu \to 0} \alpha = \frac{8.686}{c} \sqrt{\omega_p^2 - \omega^2} \ \text{db/cm}, \quad \text{if } \omega < \omega_p \qquad \text{eq. 89}$$

When $\omega_p \gg \omega$, the attenuation becomes directly proportional to the plasma frequency ($\sqrt{n}$):

$$\lim_{\nu \to 0} \alpha = 2.895 \times 10^{-10} \omega_p \ \text{db/cm}, \quad \text{if } \omega \ll \omega_p \qquad \text{eq. 90}$$

Graphic representations of eq. 80 appear in the literature [3,8,9,10,11] in normalized form generally. In this report the specific ranges of the variables were dictated by the ranges most likely to be found during reentry. Graph 1 gives the attenuation at low collision frequencies for varying plasma frequencies. Here, $\omega < \omega_p$, and the fact that the attenuation is almost independent of collision frequency is predicted by eq. 89. For higher collision frequencies, Graph 2, we see that the attenuation decreases towards zero for all plasma frequencies, a direct consequence of eq. 84.

Graph 5 is a plot of the attenuation versus plasma frequency for varying collision frequencies. In the vicinity of $\omega_p = \omega$ and for $\omega_p < \omega$, the full complexity of eq. 80 displays itself. For large ω_p, however, the attenuation becomes directly proportional to ω_p for a given collision frequency, Graph 6. Increasing ν lowers the attenuation; for low ν, the curves converge to the limiting straight line defined in eq. 90.

* * *

2. Uniform longitudinal magnetic field.

If a magnetic field threads the plasma, the conductivity becomes anisotropic as discussed earlier. If the incident wave is linearly polarized to the left of $z = 0$ in Fig. 2, it splits into two circularly rotating waves upon entering the plasma, one right hand (+), the other left hand (-) polarized. Therefore, σ must be replaced by $\sigma_\pm$ of Part I, eqs. 77. Hence,

$$\alpha_\pm = \frac{\omega}{c\sqrt{2}}\left[-1 + \frac{ne^2}{\varepsilon_0 m}\cdot\frac{(\omega \mp \omega_B)}{\omega[\nu^2+(\omega\mp\omega_B)^2]} + \sqrt{1-2\frac{ne^2}{\varepsilon_0 m}\frac{(\omega\mp\omega_B)}{\omega[\nu^2+(\omega\mp\omega_B)^2]}+\left(\frac{ne^2}{\varepsilon_0 m}\right)^2\cdot\frac{1}{\omega^2[\nu^2+(\omega\mp\omega_B)^2]}}\,\right]^{\frac{1}{2}} \qquad \text{eq. 91}$$

In terms of ω_p, this can be written

$$\alpha_\pm = 8.686\,\frac{\omega}{c\sqrt{2}}\left[-1 + \frac{(\omega\mp\omega_B)\omega_p^2}{\omega[\nu^2+(\omega\mp\omega_B)^2]} + \sqrt{1+\frac{\omega_p^4-2\omega(\omega\mp\omega_B)\omega_p^2}{\omega^2[\nu^2+(\omega\mp\omega_B)^2]}}\,\right]^{\frac{1}{2}} \text{db/cm} \qquad \text{eq. 92*}$$

Let us now consider the behavior of eq. 92 as the variables get large or small. It is eminently obvious that all the conclusions drawn for eq. 80 apply equally well for eq. 92 for both polarizations if $\omega \gg \omega_B$; i.e.,

$$\lim_{\omega_B\to 0}\alpha_\pm = \alpha = \lim_{\omega/\omega_B\to\infty}\alpha_\pm \qquad \text{eq. 93}$$

Furthermore, if $(\omega\mp\omega_B)^2 \gg \nu^2$ (which can occur for either of the two cases, $\omega \gg \nu$, or $\omega_B \gg \nu$), then

$$\lim_{\nu\to 0}\alpha_\pm = 8.686\,\frac{\omega}{c}\sqrt{\frac{\omega_p^2}{\omega(\omega\mp\omega_B)}-1}\ \text{db/cm, if } \frac{\omega_p^2}{\omega(\omega\mp\omega_B)}>1 \qquad \text{eq. 94}$$

*See Footnote**, page 21.

The attenuation of eq. 94 would show a resonance for the right handed wave in the vicinity of $\omega = \omega_B$ and become imaginary for $\frac{\omega_p^2}{\omega(\omega \mp \omega_B)} < 1$ or whenever $\omega_B > \omega$, since this guarantees the preceeding inequality; an imaginary α is equivalent to zero attenuation, since the propagation constant, γ, is pure imaginary, as was shown in eq. 88. The left handed wave would decrease monotonically with increasing ω_B for small ν. Therefore, for both waves we can say that

$$\lim_{\nu \to 0} \alpha_\pm = \lim_{\omega_B \to \infty} \alpha_\pm = 0, \quad \text{if} \quad \frac{\omega_p^2}{\omega(\omega \mp \omega_B)} < 1 \qquad \text{eq. 95}$$

If ν is very large, we have

$$\lim_{\nu \to \infty} \alpha_\pm = \lim_{\nu \to \infty} \alpha = 0 \qquad \text{eq. 96}$$

In Graphs 3 and 4, we consider the attenuation in the presence of a magnetic field of 569 gauss corresponding to a gyrofrequency of 10^{10} radians/sec. For low collision frequencies, Graph 3, we see that the attenuation has been sizably reduced as predicted by eq. 95. The effect of the magnetic field on attenuation for the higher collision frequencies, Graph 4, is very much reduced as expected from eq. 96.

Graphs 7a through 7e give the variation of attenuation with gyrofrequency, ω_p and ν fixed. Raising or lowering the plasma frequency raises or lowers the curves but does not greatly alter their form in the regions of interest. Therefore, we choose the value $\omega_p = 10^{11}$ to be representative of the curve shape. For low collision frequencies, 7a, the resonance about $\omega = \omega_B$ appears for the right handed wave as expected. The left handed wave does not resonate but decreases gradually, but would resonate about an ion ω_B' if ionic motion were included in the equations.

If the greatest reduction in attenuation is desired, then a maximum possible 3 db gain is realized if the incident wave is right circularly polarized and if the gyrofrequency is greater than the signal frequency. This is also true when the collision frequency is not small. For large ν, however, the difference between the two polarizations becomes negligible. As the collision frequency increases, a larger magnetic field is required for given decreases in attenuation. When $\nu = 10^{7}$, a magnetic field of 569 gauss ($\omega_B = 10^{10}$) reduces the attenuation of the right handed wave from 28.95 db/cm to .0072 db/cm if the plasma contains 3.2×10^{12} electrons/cm^3 ($\omega_p = 10^{11}$). This is a reduction of 99.98%! If $\nu = 10^{12}$, however, the attenuation is reduced from .738 db/cm to .734 db/cm, all remaining factors being equal. Even if the field is increased to 5,690 gauss ($\omega_B = 10^{11}$) at this collision frequency, the attenuation is only reduced to .699 db/cm, a decrease of 5%. (Note that in the practical case of transmitting through the plasma, nature has been quite helpful, for the magnetic field is most effective in the regions of high attenuation and least effective where the attenuation is already small).

* * *

B. Reflection and Transmission

1. Transmission Line Analogy

Let us briefly review the governing equations for transmission lines.[12] Consider Fig. 3 for the case of the lossless line:

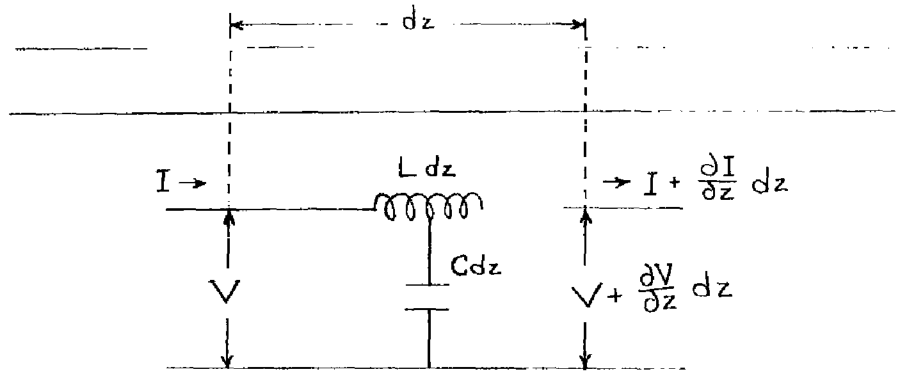

FIG. 3

We can immediately write

$$V = L\frac{dI}{dt} + RI + C\int I\,dt \qquad \text{eq. 97}$$

$$\frac{\partial V}{dz}\,dz = -L\,dz\,\frac{\partial I}{\partial t} \qquad \text{eq. 98a}$$

$$\frac{\partial I}{\partial z}\,dz = -C\,dz\,\frac{\partial V}{\partial t} \qquad \text{eq. 98b}$$

Therefore,

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t} \qquad \text{eq. 99a}$$

$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t} \qquad \text{eq. 99b}$$

Eliminating the current, I, from eqs. 99 yields

$$\boxed{\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2}} \qquad \text{eq. 100}$$

But LC has the dimensions of $1/v^2$. Eq. 100, then, has the exact same form as the free space wave equation, (v = velocity of propagation)

$$\frac{\partial^2 E}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \qquad \text{eq. 101}$$

for a wave traveling through free space in the z direction, if we let

$$LC = \frac{1}{c^2} = \mu_0 \epsilon_0 \qquad \text{eq. 102}$$

Now

$$Z_0 \equiv Lv = \sqrt{\frac{L}{C}} \qquad \text{eq. 103}$$

is defined as the characteristic impedance of the line. If the terminating or load impedance is Z_l , then

$$\rho = \frac{V_{ref}}{V_{inc}} = \frac{Z_l - Z_0}{Z_l + Z_0} \text{ (reflection coefficient)} \qquad \text{eq. 104a}$$

$$\tau = \frac{V_{tran.}}{V_{inc.}} = \frac{2Z_l}{Z_l + Z_0} \text{ (transmission coefficient)} \qquad \text{eq. 104b}$$

Let us consider the sinusoidal variation of voltage and current in time and space, where the time dependence is always understood to be $e^{j\omega t}$:

$$V = V_1 e^{-j\psi z} + V_1' e^{j\psi z} \qquad \text{eq. 105a}$$

$$I = \frac{1}{Z_0}\left[V_1 e^{-j\psi z} - V_1' e^{j\psi z}\right] \qquad \text{eq. 105b}$$

$$\psi = \omega\sqrt{LC} \qquad \text{eq. 105c}$$

where V_1 and V_1' are the incident and reflected voltages respectively. At z = 0, the terminus of the line, we have

$$Z_l \equiv \frac{V}{I} = \frac{V_1 + V_1'}{V_1 - V_1'} \cdot Z_0 \qquad \text{eq. 106}$$

Therefore,

$$\frac{V_1'}{V_1} = \frac{Z_l - Z_0}{Z_l + Z_0} = \rho \qquad \text{eq. 107}$$

as in eq. 104a. At $z = -l$,

$$Z_i = \frac{Z_l \cos\psi l + jZ_0 \sin\psi l}{Z_0 \cos\psi l + jZ_0 \sin\psi l} \qquad \text{eq. 108}$$

Now we want to show that the results of transmission line theory hold for reflection from and transmission through a plasma slab. To the left of z = 0 in Fig. 2, we have for an electric wave linearly polarized in the x-direction:

$$E_x(z) = E e^{-jk_0 z} + E' e^{jk_0 z} \qquad \text{eq. 109a}$$

and for the magnetic field in the y-direction:

$$H_y(z) = \frac{1}{\eta_0}\left[E e^{-jk_0 z} - E' e^{jk_0 z}\right] \qquad \text{eq. 109b}$$

where E and E' are the incident and reflected amplitudes and

$$k_0 = \omega\sqrt{\mu_0 \epsilon_0} \qquad \text{eq. 109c}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \qquad \text{eq. 109d}$$

Therefore the analogy is complete if eqs. 109 and 105 hold and if we define

$$Z(z) = \frac{E_x(z)}{H_y(z)} \qquad \text{eq. 110}$$

In the general case, if 1 and 2 are two different media, we see, from Fig. 4, that

$$\rho = \frac{E_1'}{E_1} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \text{eq. 111}$$

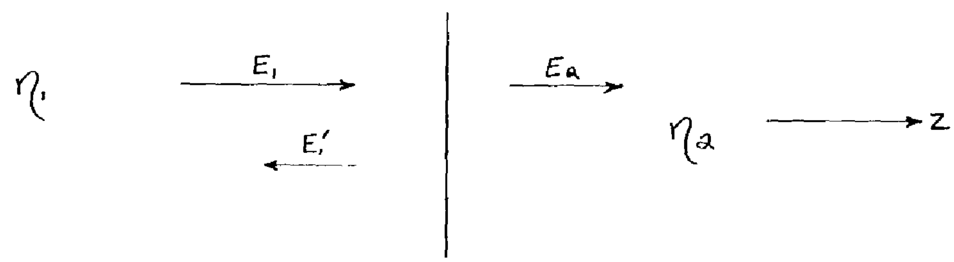

FIG. 4

If there are three media, Fig. 5, then

$$Z_{\ell_1} = Z_{i2} = \eta_2 \left[\frac{\eta_3 \cos k_2 \ell + j \eta_2 \sin k_2 \ell}{\eta_2 \cos k_2 \ell + j \eta_3 \sin k_2 \ell} \right] \qquad \text{eq. 112}$$

and

$$\rho = \frac{Z_{\ell_1} - \eta_1}{Z_{\ell_1} + \eta_1} \qquad \text{eq. 113}$$

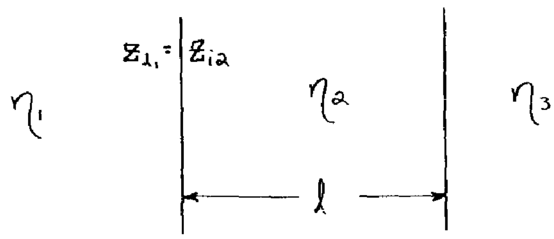

FIG. 5

Now let us consider the plasma to be a lossy dielectric (or an imperfect conductor). We have already shown that

$$\varepsilon = 1 - j \frac{\sigma}{\omega \varepsilon_0} \qquad \text{eq. 20}$$

$$\gamma = jk = j \frac{\omega}{c} \sqrt{1 - j \frac{\sigma}{\omega \varepsilon_0}} = \alpha + j\beta \qquad \text{eq. 27}$$

Therefore,

$$\eta_p = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{1}{1 - j\sigma/\omega\varepsilon_0}} \qquad \text{eq. 114}$$

and

$$\rho = \frac{\eta_p - \eta_0}{\eta_p + \eta_0} \qquad \text{eq. 115}$$

2. Plasma Reflection and Transmission

a. Without a magnetic field.

Consider the general case of a wave incident on a semi-infinite plasma at an angle Θ, Fig. 6.

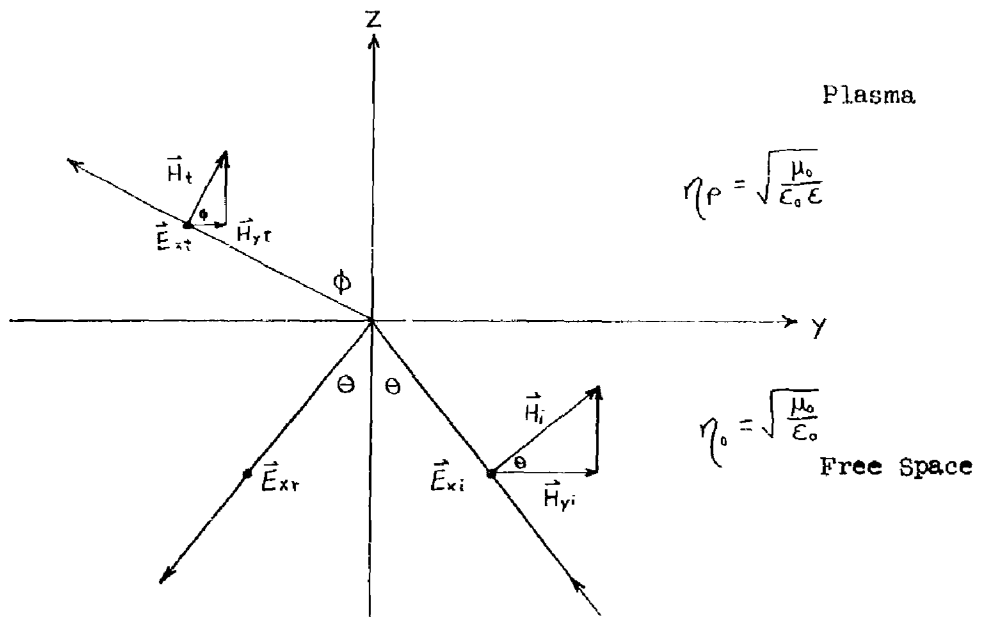

FIG. 6

From eq. 110, we have

$$Z_i \equiv \frac{E_{xi}}{H_{yi}} = \frac{E_{xi}}{H_i \cos\theta} = \eta_0 \sec\theta \qquad \text{eq. 116a}$$

$$Z_\ell \equiv \frac{E_{xt}}{H_{yt}} = \eta_p \sec\theta \qquad \text{eq. 116b}$$

Now

$$\rho = \frac{Z_\ell - Z_i}{Z_\ell + Z_i} = \frac{1 - Z_i/Z_\ell}{1 + Z_i/Z_\ell} \qquad \text{eq. 104a}$$

But

$$\frac{Z_i}{Z_\ell} = \frac{\eta_0 \sec\theta}{\eta_p \sec\phi} = \sqrt{\varepsilon}\,\frac{\cos\phi}{\cos\theta} \qquad \text{eq. 117}$$

and the index of refraction given in Part I requires

$$N^2 = \frac{\sin^2\theta}{\sin^2\phi} = \varepsilon = \frac{\sin^2\theta}{1 - \cos^2\phi} \qquad \text{eq. 118}$$

which implies that

$$\cos\phi = \frac{\sqrt{\varepsilon - \sin^2\theta}}{\sqrt{\varepsilon}} \qquad \text{eq. 119}$$

and

$$\frac{Z_1}{Z_{11}} = \frac{\sqrt{\varepsilon - \sin^2\theta}}{\cos\theta} \qquad \text{eq. 120}$$

The reflectior coefficient, ρ , is then,

$$\rho = \frac{1 - \sqrt{\varepsilon - \sin^2\theta} \;/\cos\theta}{1 + \sqrt{\varepsilon - \sin^2\theta} \;/\cos\theta} \qquad \text{eq. 121}$$

$$\boxed{\rho = \frac{E_r}{E_i} = \frac{\cos\theta - \sqrt{\varepsilon - \sin^2\theta}}{\cos\theta + \sqrt{\varepsilon - \sin^2\theta}}} \qquad \text{eq. 122}$$

For the special case of normal incidence, we have

$$\rho = \frac{1 - \sqrt{\varepsilon}}{1 + \sqrt{\varepsilon}} \qquad \text{eq. 123}$$

For a further discussion of plasma reflection, see[13,14,15].

The ratio of reflected to incident power, R, for normal incidence ($\theta = 0$) is given by

$$R = \frac{P_{ref}}{P_{inc}} = \left|\rho\right|^2 = \left|\frac{1 - \sqrt{\varepsilon}}{1 + \sqrt{\varepsilon}}\right|^2 \qquad \text{eq. 124}$$

But

$$\sqrt{\varepsilon} = -j\frac{1}{K_o}\gamma = -j\frac{1}{K_o}(\alpha + j\beta) = \frac{\beta}{K_o} - j\frac{\alpha}{K_o} \qquad \text{eq. 26}$$

Therefore,

$$R = \frac{(1 - \beta/K_o)^2 + (\alpha/K_o)^2}{(1 + \beta/K_o)^2 + (\alpha/K_o)^2} \qquad \text{eq. 125}$$

$$\boxed{R = \frac{(K_o - \beta)^2 + \alpha^2}{(K_o + \beta)^2 + \alpha^2}} \qquad \text{eq. 126*}$$

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*See Footnote**, page 21

where α is given by eq. 80 and β is defined by eq. 34. β can also be written in the form of eq. 32.

$$\beta^2 = \alpha^2 + k_o^2 \epsilon_r = \alpha^2 + k_o^2 \left(1 - \frac{\omega_p^2}{\omega^2 + \nu^2}\right) \qquad \text{eq. 127}$$

From the conservation of energy, we know that

$$T = \frac{P_{tran}}{P_{inc}} = 1 - R \qquad \text{eq. 128}$$

Therefore,

$$\boxed{T = \frac{4k_o\beta}{(K_o+\beta)^2 + \alpha^2}} \qquad \text{eq. 129*}$$

and

$$T(-db) = 10 \log_{10} \frac{4k_o\beta}{(K_o+\beta)^2 + \alpha^2} \qquad \text{eq. 130*}$$

We can now investigate the limiting cases of eqs. 126 and 129 with the knowledge of the behavior of α given in Section A.1. To summarize, we have

$$\lim_{\omega\to\infty} \alpha = \lim_{\nu\to\infty} \alpha = 0 \qquad \text{eq. 83-84}$$

$$\lim_{\nu\to 0} \alpha = 0, \text{ if } \omega \geq \omega_p \qquad \text{eq. 88}$$

$$\lim_{\nu\to 0} \alpha = \frac{1}{c}\sqrt{\omega_p^2 - \omega^2}, \text{ if } \omega < \omega_p \qquad \text{eq. 89}$$

$$\lim_{\nu\to 0} \alpha = \frac{\omega_p}{c}, \text{ if } \omega \ll \omega_p \qquad \text{eq. 90}$$

From eq. 127 and the above, we get

$$\lim_{\omega\to\infty} \beta = \lim_{\nu\to\infty} \beta = K_o = \frac{\omega}{c} \qquad \text{eq. 131a}$$

$$\lim_{\nu\to 0} \beta = \frac{1}{c}\sqrt{\omega^2 - \omega_p^2}, \text{ if } \omega > \omega_p \qquad \text{eq. 131b}$$

$$\lim_{\nu\to 0} \beta = 0, \text{ if } \omega \leq \omega_p \qquad \text{eq. 131c}$$

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*See Footnote **, page 21

Applying the above limits to eq. 129 yields

$$\lim_{\omega \to \infty} T = \lim_{\nu \to \infty} T = 1 \qquad \text{eq. 132a}$$

$$\lim_{\nu \to 0} T = \frac{4\sqrt{1-\omega_p^2/\omega^2}}{(1+\sqrt{1-\omega_p^2/\omega^2})^2}, \text{ if } \omega > \omega_p \qquad \text{eq. 132b}$$

$$\lim_{\nu \to 0} T = 1, \text{ if } \omega \gg \omega_p \qquad \text{eq. 132c}$$

$$\lim_{\nu \to 0} T = 0, \text{ if } \omega \le \omega_p \qquad \text{eq. 132d}$$

If ω gets very small, it can easily be shown that

$$\lim_{\omega \to 0} \alpha = 0 = \lim_{\omega \to 0} \beta \qquad \text{eq. 133}$$

Consequently,

$$\lim_{\omega \to 0} T = 0 \qquad \text{eq. 134}$$

i.e., even though the attenuation is decreasing, we still have a severe loss problem due to increasing reflection. For this reason, small signal frequencies were not discussed in Section A as they would have led to erroneous conclusions. For large ω or ν, however, we see that the transmission increases simultaneously with decreasing attenuation, thereby greatly enhancing the penetration of the plasma. Similarly, for small ν, the transmission increases while the attenuation decreases when $\omega > \omega_p$. The plasma becomes more opaque if the converse holds, $\omega < \omega_p$.

Graphs 8 and 9 are logarithmic (-db) plots of the transmission. Since $\log_{10} 1 = 0$, we see that eq. 132a is verified, since increasing ν pushes the curves down towards zero. As ν gets small, the losses increase, $\log_{10} T \to -\infty$, when $\omega_p > \omega$. When $\omega_p < \omega$, the losses drop rapidly towards zero and become independent of ν as can be seen from eq. 132b and c.

* *

b. Uniform longitudinal magnetic field.

When a constant magnetic field laces the plasma, the attenuation and phase constants are altered due to the change in the conductivity tensor; i.e.,

$$\alpha \xrightarrow{B = \text{constant}} \alpha_{\pm}$$

$$\beta \xrightarrow{B = \text{constant}} \beta_{\pm}$$

$\alpha_{\pm}$ is given by eq. 92 and $\beta_{\pm}$ is determined by combining eq. 32 with eq. 77b:

$$\beta_{\pm}^{2} = \alpha_{\pm}^{2} + k_0^{2}\left[1 - \frac{\omega_p^{2}(\omega \mp \omega_B)}{\omega[\nu^2 + (\omega \mp \omega_B)^2]}\right] \qquad \text{eq. 135}$$

The reflection and transmission coefficients for the two waves are then specified by direct substitution into eq. 126 and 129:

$$R_{\pm} = \frac{(k_0 - \beta_{\pm})^2 + \alpha_{\pm}^2}{(k_0 + \beta_{\pm})^2 + \alpha_{\pm}^2} \qquad \text{eq. 136*}$$

$$T_{\pm} = \frac{4 k_0 \beta_{\pm}}{(k_0 + \beta_{\pm})^2 + \alpha_{\pm}^2} \qquad \text{eq. 137*}$$

From Section A.2, recall that

$$\lim_{\omega_B \to 0} \alpha_{\pm} = \alpha = \lim_{\omega/\omega_B \to \infty} \alpha_{\pm} \qquad \text{eq. 93}$$

$$\lim_{\nu \to \infty} \alpha_{\pm} = \lim_{\nu \to \infty} \alpha = 0 \qquad \text{eq. 96}$$

$$\lim_{\nu \to 0} \alpha_{\pm} = \lim_{\omega_B \to \infty} \alpha_{\pm} = 0, \quad \text{if} \quad \frac{\omega_p^2}{\omega(\omega \mp \omega_B)} < 1 \qquad \text{eq. 95}$$

---

* See Footnote **, p. 21

$$\lim_{\nu \to 0} \alpha_{\pm} = \frac{\omega}{c}\sqrt{\frac{\omega_p^2}{\omega(\omega \mp \omega_B)} - 1}, \quad \text{if } \frac{\omega_p^2}{\omega(\omega \mp \omega_B)} < 1$$ eq. 94

From eq. 135 and the above, we get

$$\lim_{\omega_B \to 0} \beta_{\pm} = \beta = \lim_{\omega/\omega_B \to \infty} \beta_{\pm}$$ eq. 138a

$$\lim_{\nu \to \infty} \beta_{\pm} = \lim_{\nu \to \infty} \beta = \frac{\omega}{c}$$ eq. 138b

$$\lim_{\omega_B \to \infty} \beta_{\pm} = \frac{\omega}{c}$$ eq. 138c

$$\lim_{\nu \to 0} \beta_{\pm} = \frac{\omega}{c}\sqrt{1 - \frac{\omega_p^2}{\omega(\omega \mp \omega_B)}}, \quad \text{if } \frac{\omega_p^2}{\omega(\omega \mp \omega_B)} < 1$$ eq. 138d

$$\lim_{\nu \to 0} \beta_{\pm} = 0, \quad \text{if } \frac{\omega_p^2}{\omega(\omega \mp \omega_B)} > 1$$ eq. 138e

Applying the above to eq. 137 yields

$$\lim_{\omega_B \to 0} T_{\pm} = \lim_{\omega/\omega_B \to \infty} T_{\pm} = T$$ eq. 139a

$$\lim_{\nu \to \infty} T_{\pm} = \lim_{\nu \to \infty} T = 1$$ eq. 139b

$$\lim_{\omega_B \to \infty} T_{\pm} = 1$$ eq. 139c

$$\lim_{\nu \to 0} T_{\pm} = \frac{4\sqrt{1 - \frac{\omega_p^2}{\omega(\omega \mp \omega_B)}}}{\left(1 + \sqrt{1 - \frac{\omega_p^2}{\omega(\omega \mp \omega_B)}}\right)^2}, \quad \text{if } \frac{\omega_p^2}{\omega(\omega \mp \omega_B)} < 1$$ eq. 139d

$$\lim_{\nu \to 0} T_{\pm} = 1, \quad \text{if } \frac{\omega_p^2}{\omega(\omega \mp \omega_B)} << 1$$ eq. 139e

$$\lim_{\nu \to 0} T_{\pm} = 0, \quad \text{if } \frac{\omega_p^2}{\omega(\omega \mp \omega_B)} > 1$$ eq. 139f

Graph 10 gives the transmission loss for a right hand polarized wave traveling through a plasma interlaced by a uniform magnetic field of 569 gauss along the direction of propagation. For low ν, the inequality of eq. 139d is guaranteed since $\omega_B > \omega$ and T becomes independent of ν. As $\nu \to \infty$, $T_\pm \to T$; over the entire range, the loss has been reduced.

Graphs 11a - 11f show the effect of the magnetic field on transmission for fixed ω_p and ν. The independence of ν is strikingly demonstrated by the convergence of T to approximately -4.15 db at $\omega_B = 10^{11}$ for ν running from $10^6 \to 10^{10}$. As with the attenuation, the right hand polarization yields the lower losses.

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APPENDIX

I. Earth's Magnetic Field.

Approximation 5, page 4 can be easily justified by considering the fact that the magnetic field almost never exceeds 0.3 gauss (unless one happens to live in the Gulf of Siam, where B = 0.4 gauss). An electron in this field rotates at a gyrofrequency of $\omega_B = 5.28\times10^6$ radians/sec. Since, for most frequencies of interest, $\omega \gg \omega_B$, the earth's magnetic field has only a very small effect on the transmission properties of the plasma.

II. Critical Field Strength for Perturbation Approximation.[1]

The effect of the microwave field on the plasma was assumed to be small in the derivation of the foregoing equations. The upper limitations can be estimated as follows:

The mean electron energy (3 degrees of freedom) is given by

$$W = 3/2\ kT$$

where

T = temperature

k = Boltzmann's constant = 1.38×10^{-23} joules/deg. mole

The time rate of change of this energy can be attributed to: 1. fixed fields and other factors, $\frac{DW}{Dt}$; 2. the presence of the microwave field, $\frac{dW}{dt}$; 3. and to collisions of the electrons, $\frac{\delta W}{\delta t}$. Then

$$\frac{\partial W}{\partial t} = \frac{DW}{Dt} + \frac{dW}{dt} + \frac{\delta W}{\delta t} \qquad \text{eq. A-1}$$

The condition that the microwave field have a small effect on W is that

$$\left|\frac{dW}{dt}\right| \ll \left|\frac{DW}{Dt}\right| + \left|\frac{\delta W}{\delta t}\right| \qquad \text{eq. A-2}$$

If $\frac{DW}{Dt}$ is neglected, the inequality is still true and eq. A-2 becomes

$$\frac{1}{2}\frac{J_r \cdot E}{n} = \frac{1}{2}\frac{\sigma_r E^2}{n} \ll \chi_e \nu W$$

where

σ_r = real part of conductivity

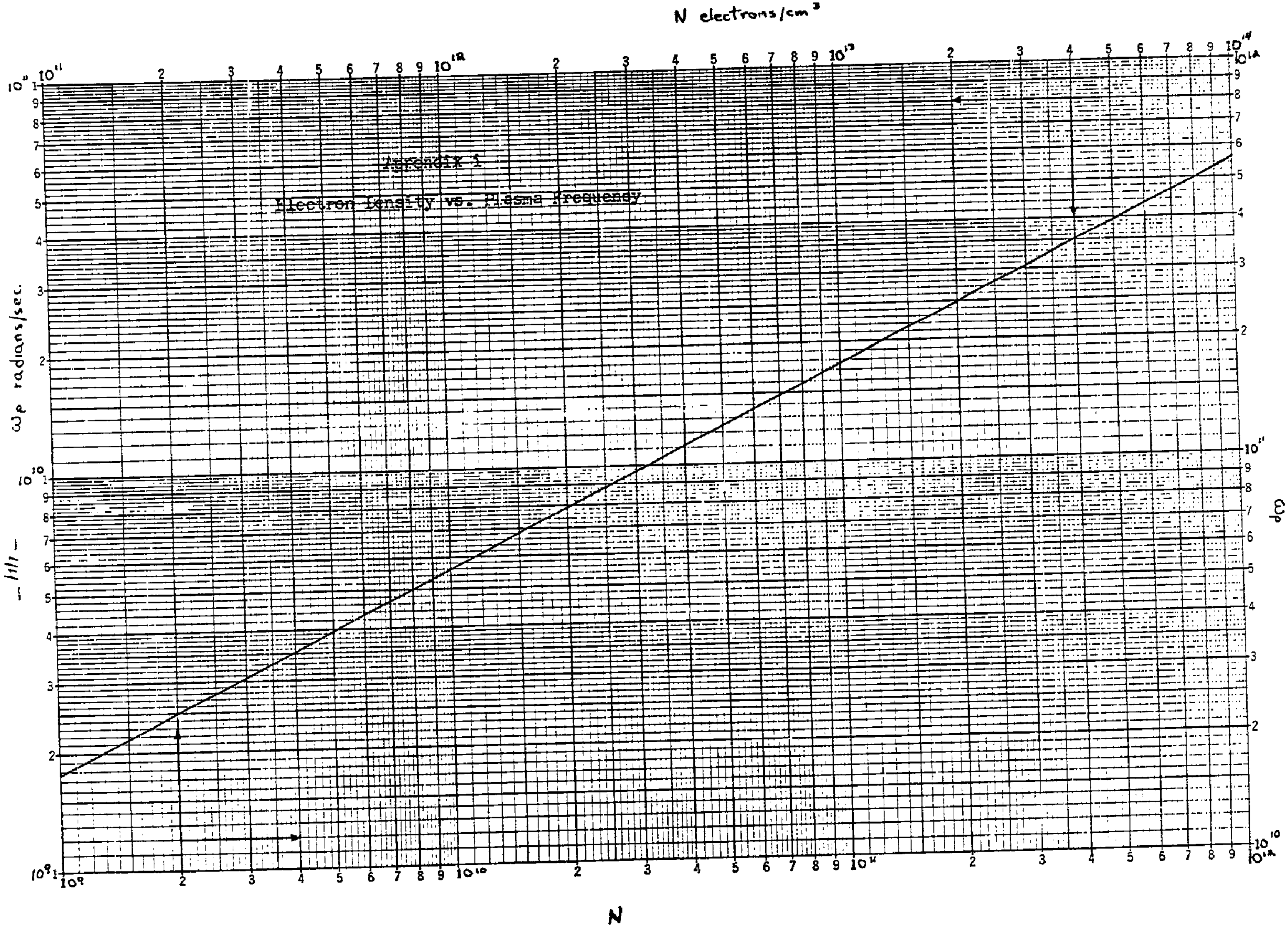
$$= \frac{ne^2}{m}\frac{\nu}{\omega^2 + \nu^2}$$

χ_e = effective energy transfer coefficient per collision.

Then the field strength limit is given by

$$E^2 \ll E_{cr}^2 = \frac{3\chi_e k T m}{e^2}(\omega^2 + \nu^2)$$

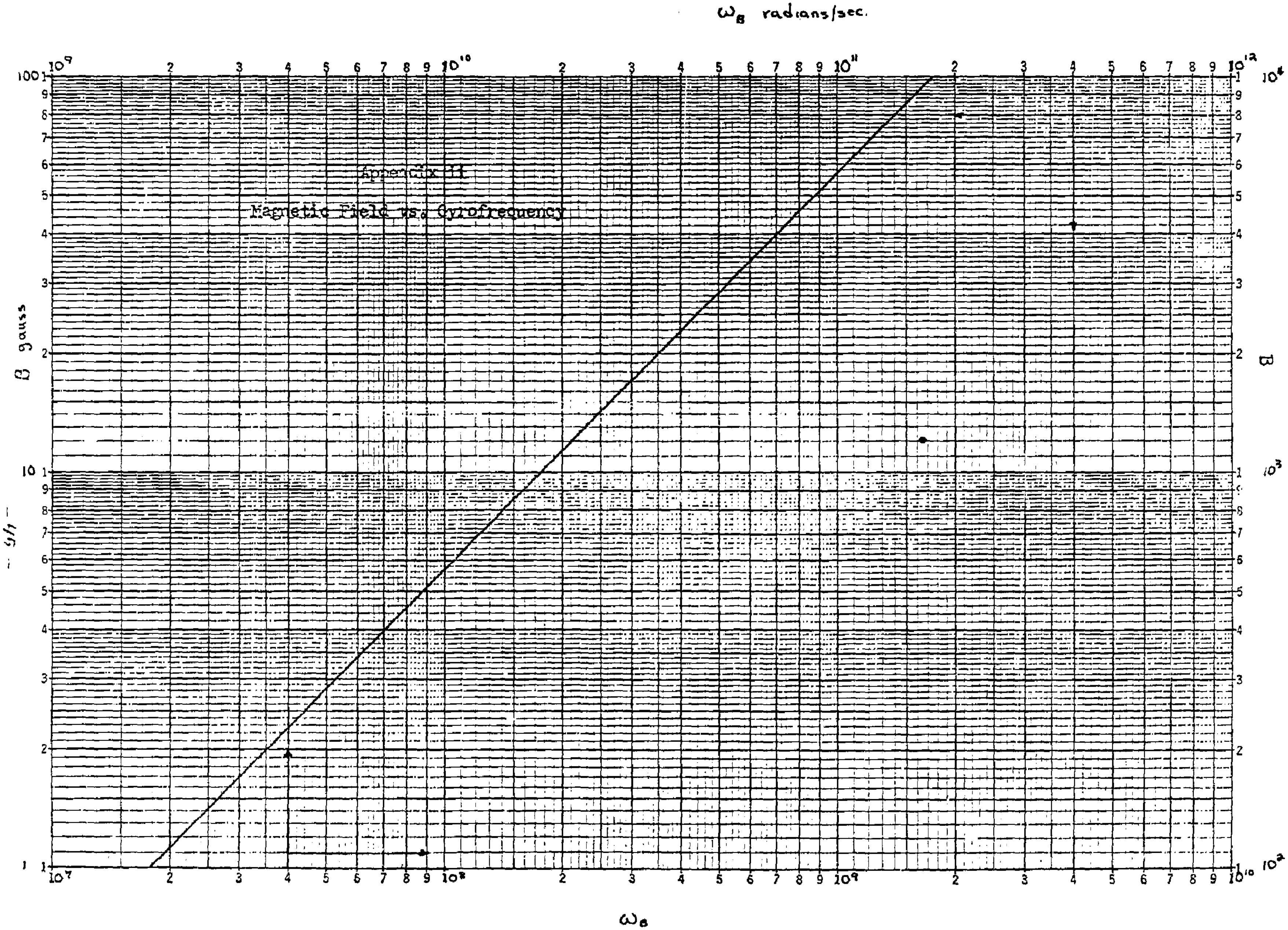
INDEX

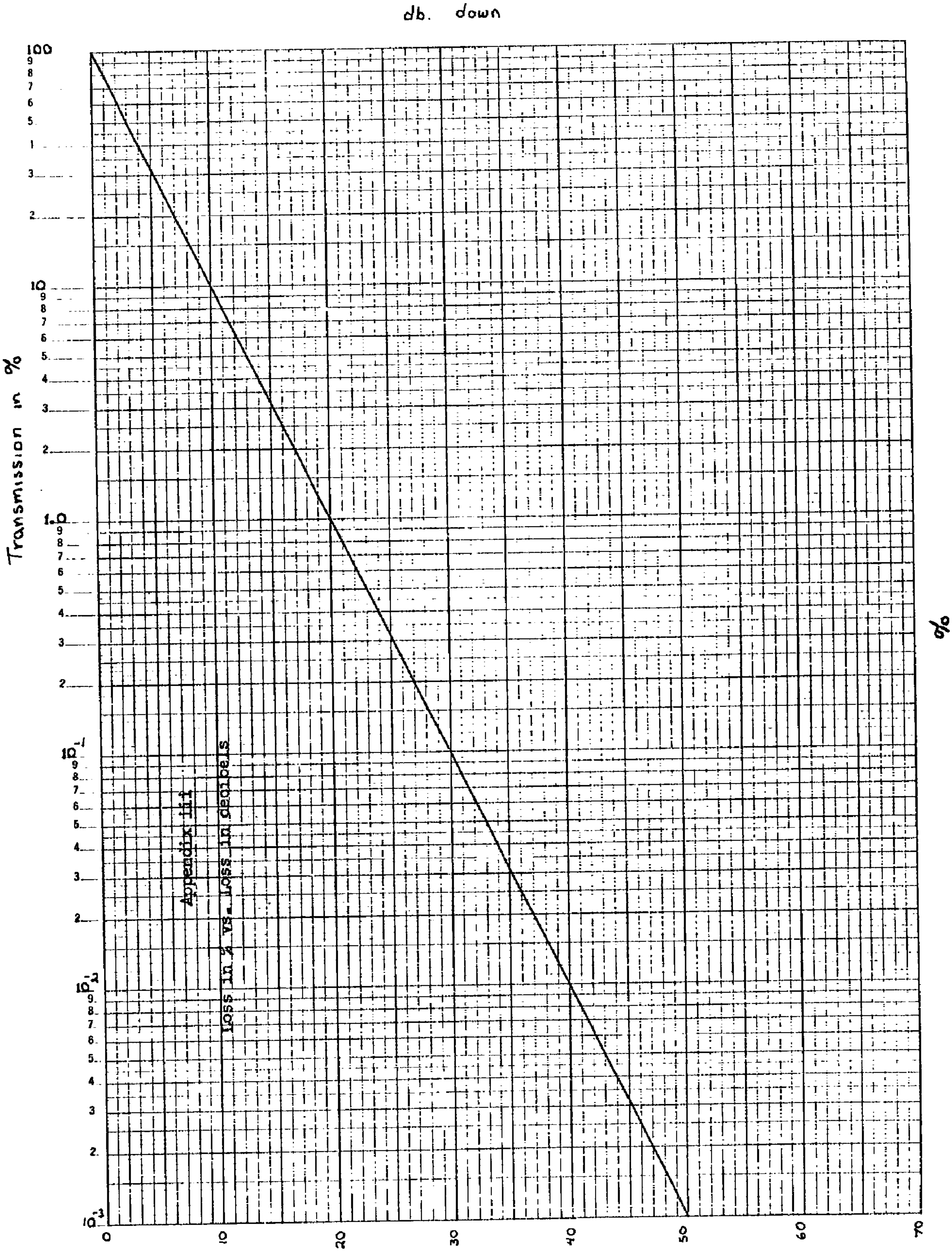


N electrons/cm³
Appendix 1
Electron Density vs. Plasma Frequency
ωp radians/sec.
ωp
N

Appendix II

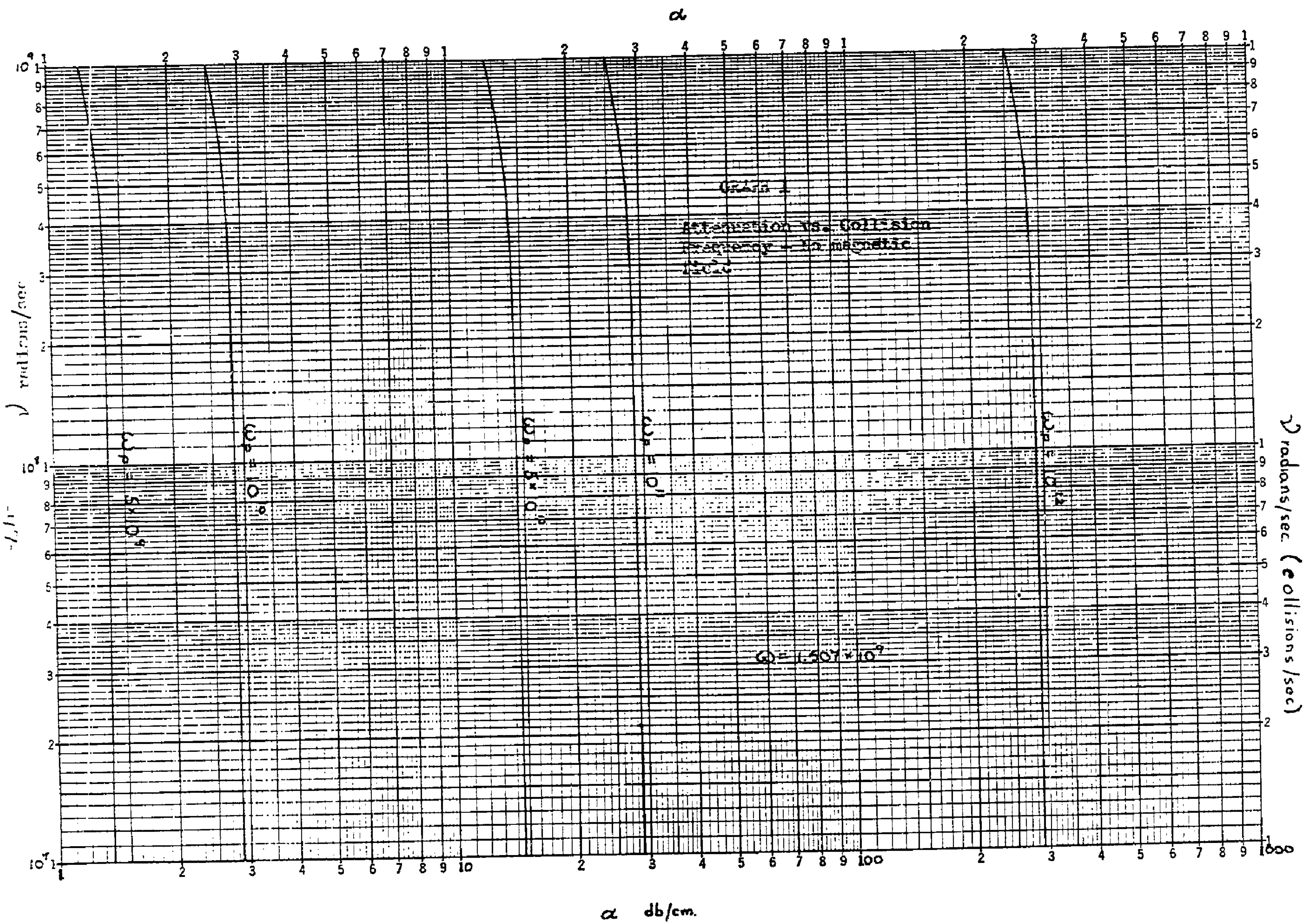
Magnetic Field vs. Gyrofrequency

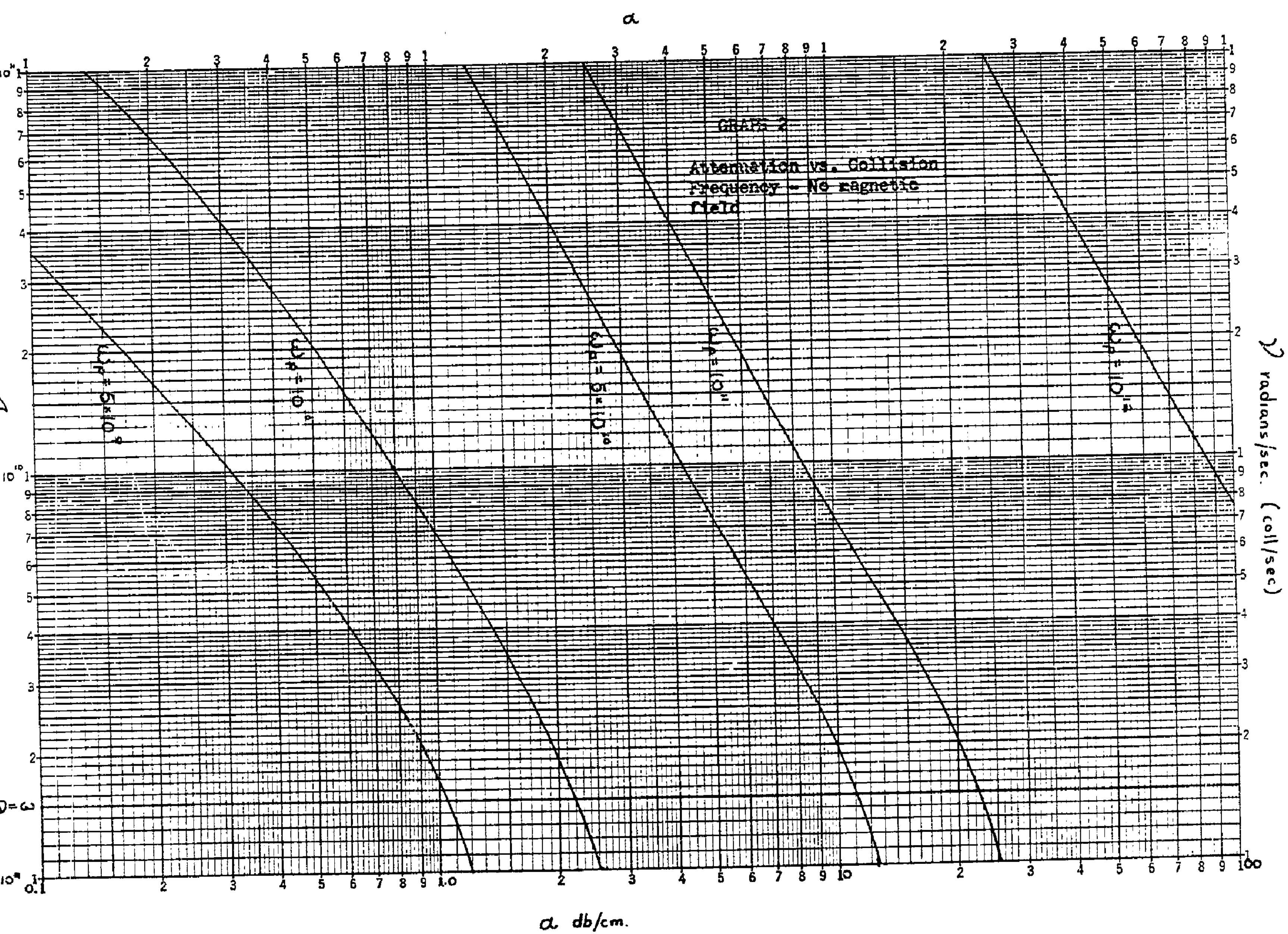


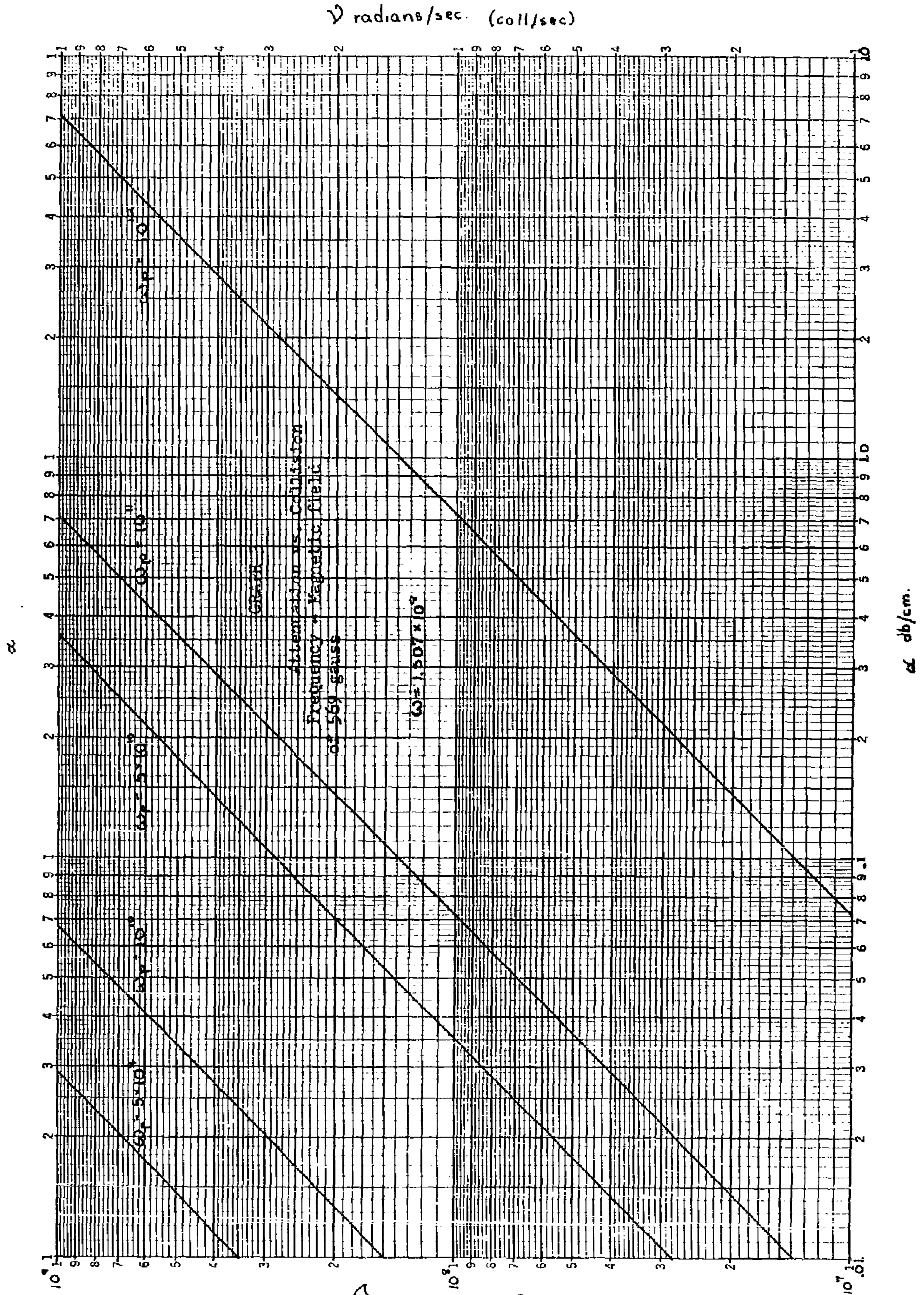


Appendix iii

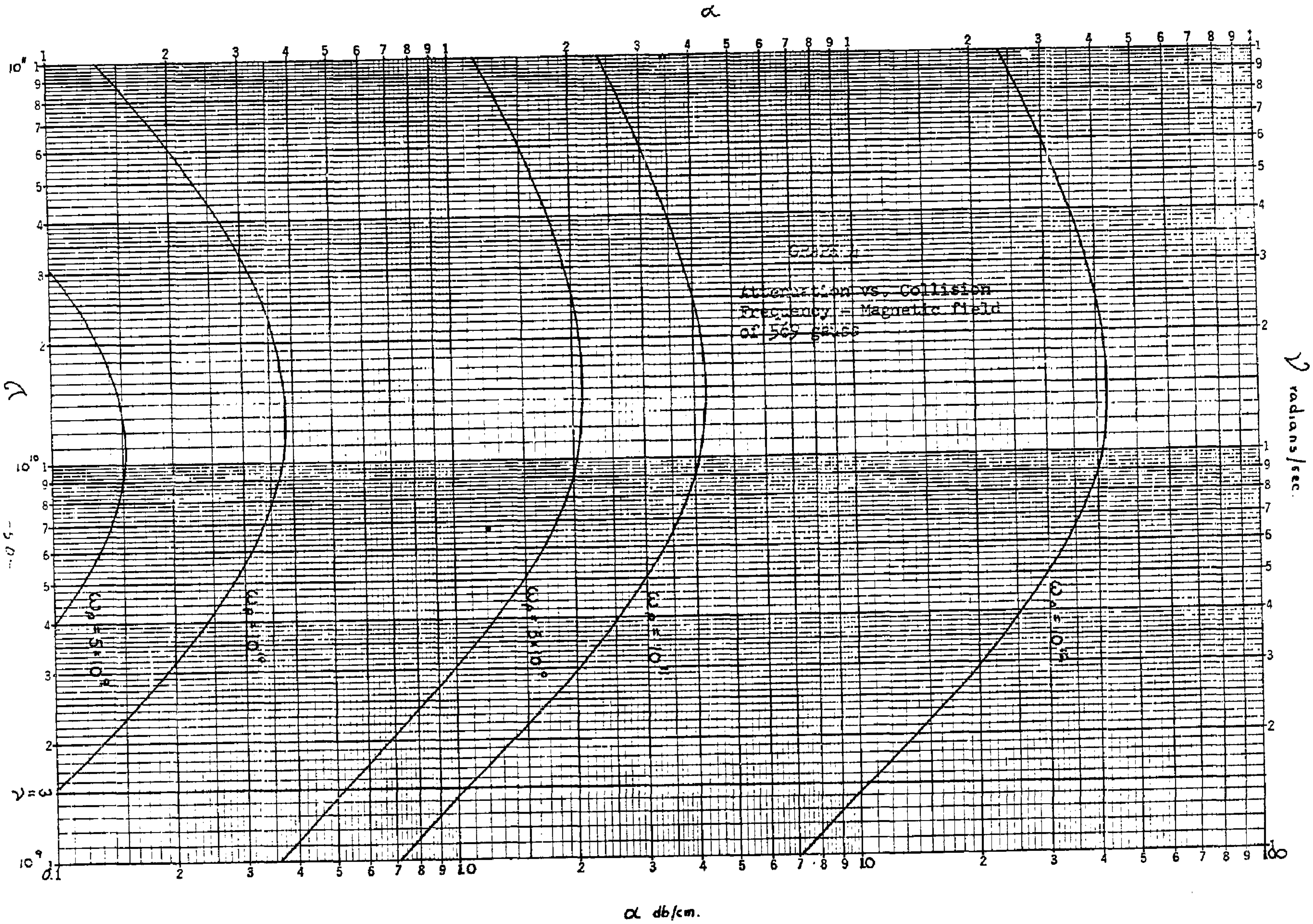
Loss in % vs. Loss in decibels


α
Graph 1
Attenuation vs. Collision
Frequency – No magnetic
field
$\omega_p = 5\times10^9$
$\omega_p = 10^{10}$
$\omega_p = 5\times10^{10}$
$\omega_p = 10^{11}$
$\omega_p = 10^{12}$
$\omega = 1.507\times10^{9}$
ν radians/sec (collisions/sec)
ν radians/sec
α db/cm.

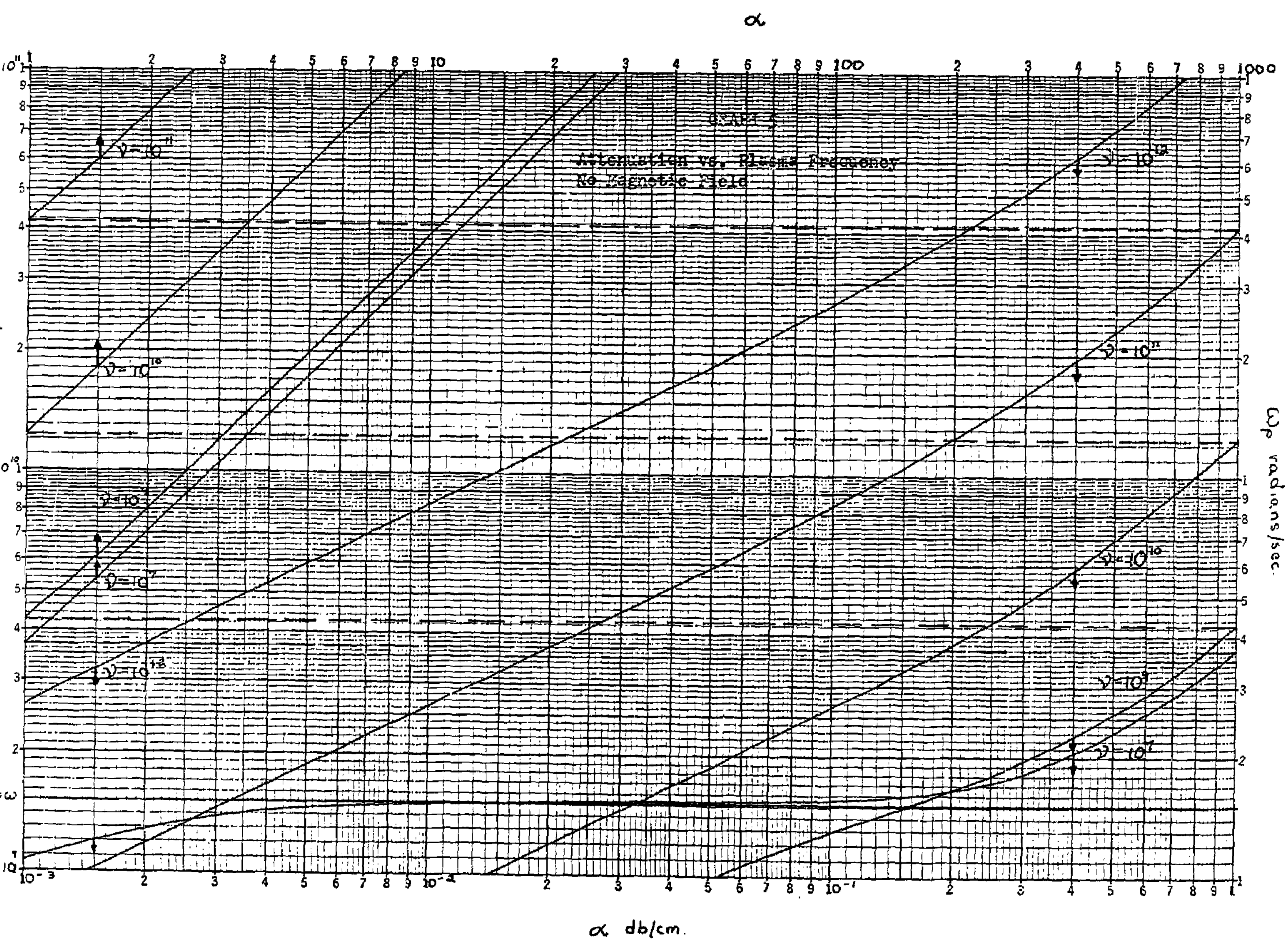

GRAPH 2
Attenuation vs. Collision Frequency - No magnetic field
α
α db/cm.
ν radians/sec. (coll/sec)
ν
$\omega_p = 5\times10^9$
$\omega_p = 10^{10}$
$\omega_p = 5\times10^{10}$
$\omega_p = 10^{11}$
$\omega_p = 10^{12}$
$\nu = \omega$

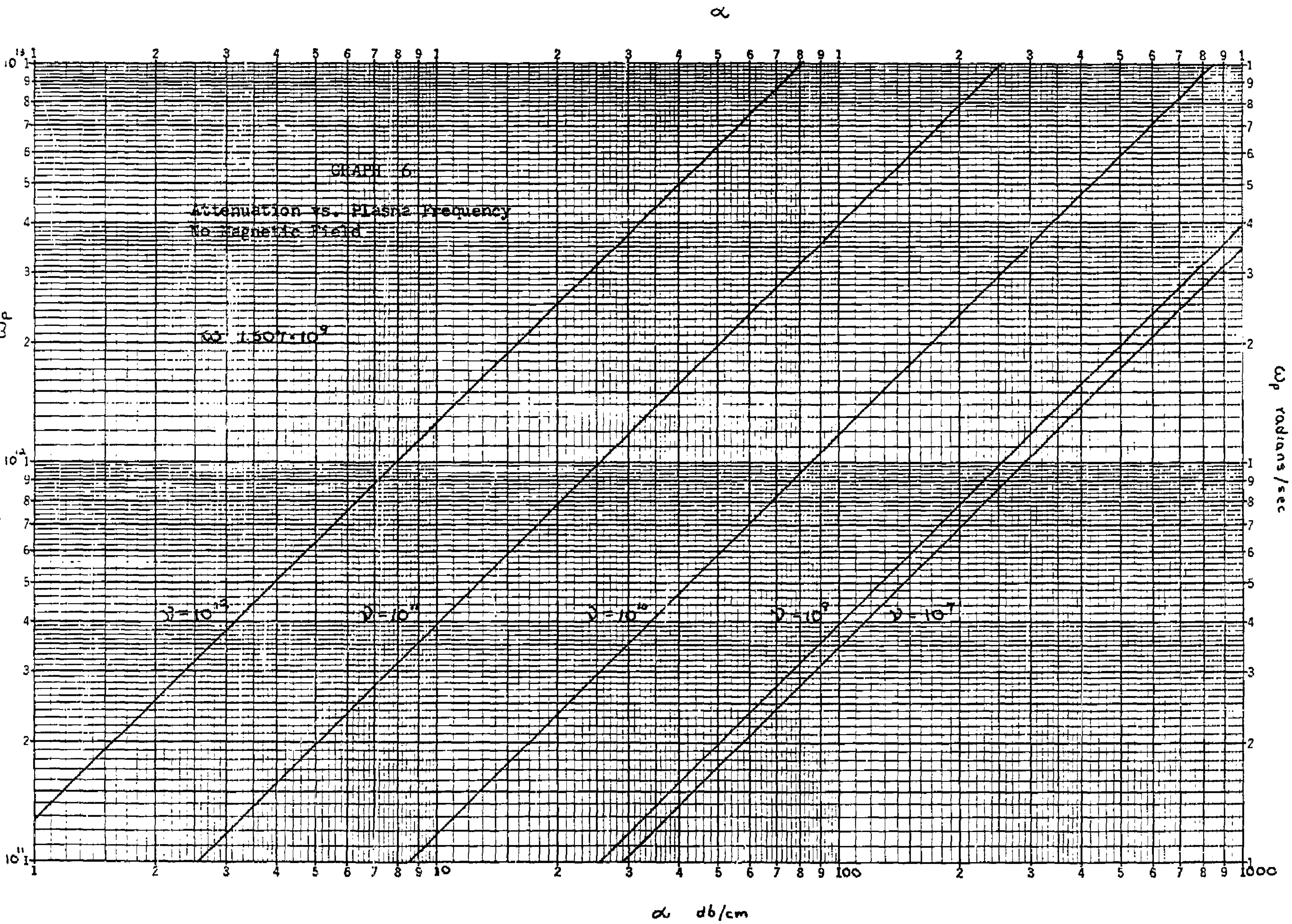


ν radians/sec. (coll/sec)
GRAPH
Attenuation vs. Collision Frequency - Magnetic Field of 559 gauss
ω = 1.507×10^9
α db/cm.
α

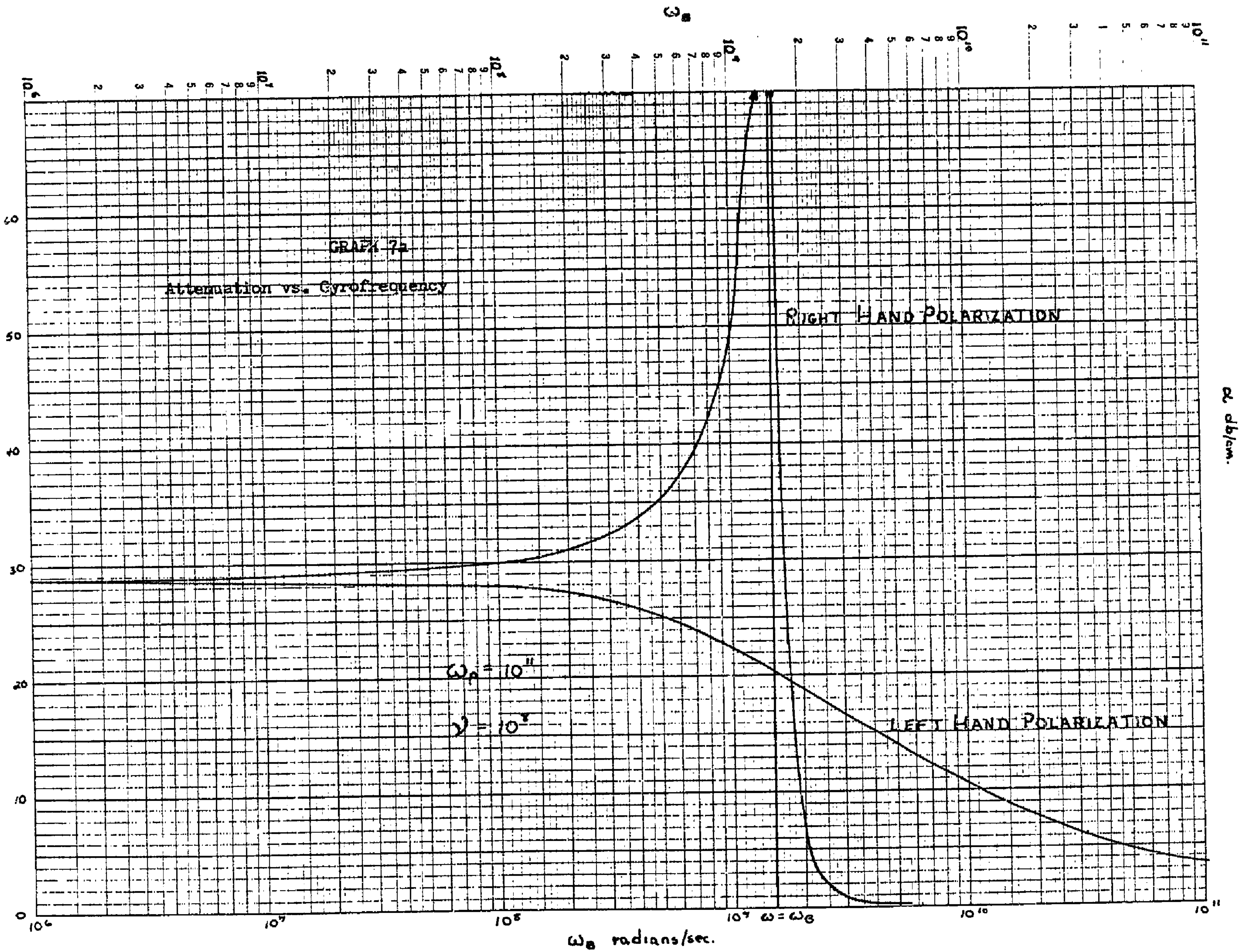


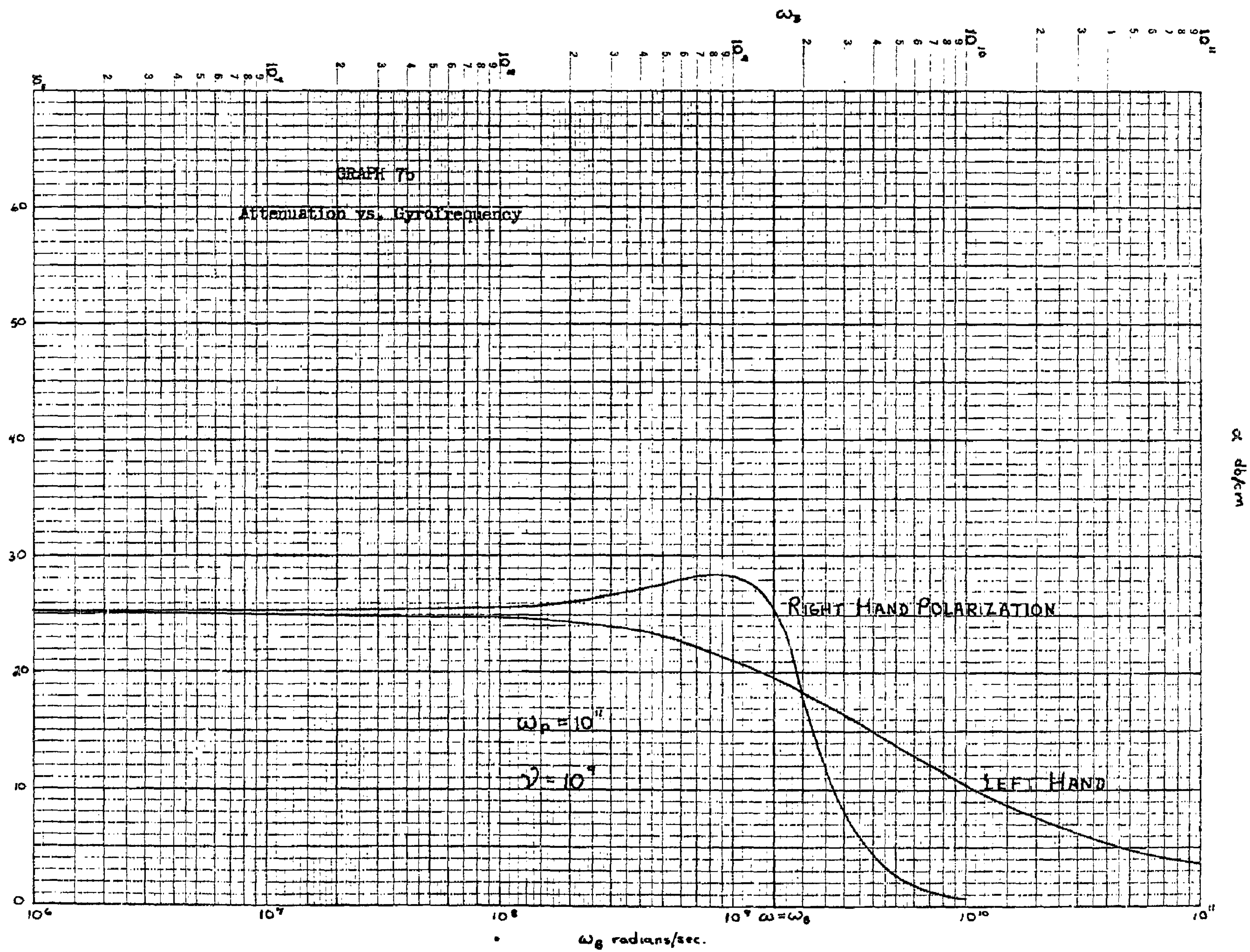
Graph
Attenuation vs. Collision Frequency - Magnetic field of 569 gauss
α
α db/cm.
ν radians/sec.
ν
$\nu = \omega$
$\omega_p = 5 \times 10^9$
$\omega_p = 10^{10}$
$\omega_p = 5 \times 10^{10}$
$\omega_p = 10^{11}$
$\omega_p = 10^{12}$

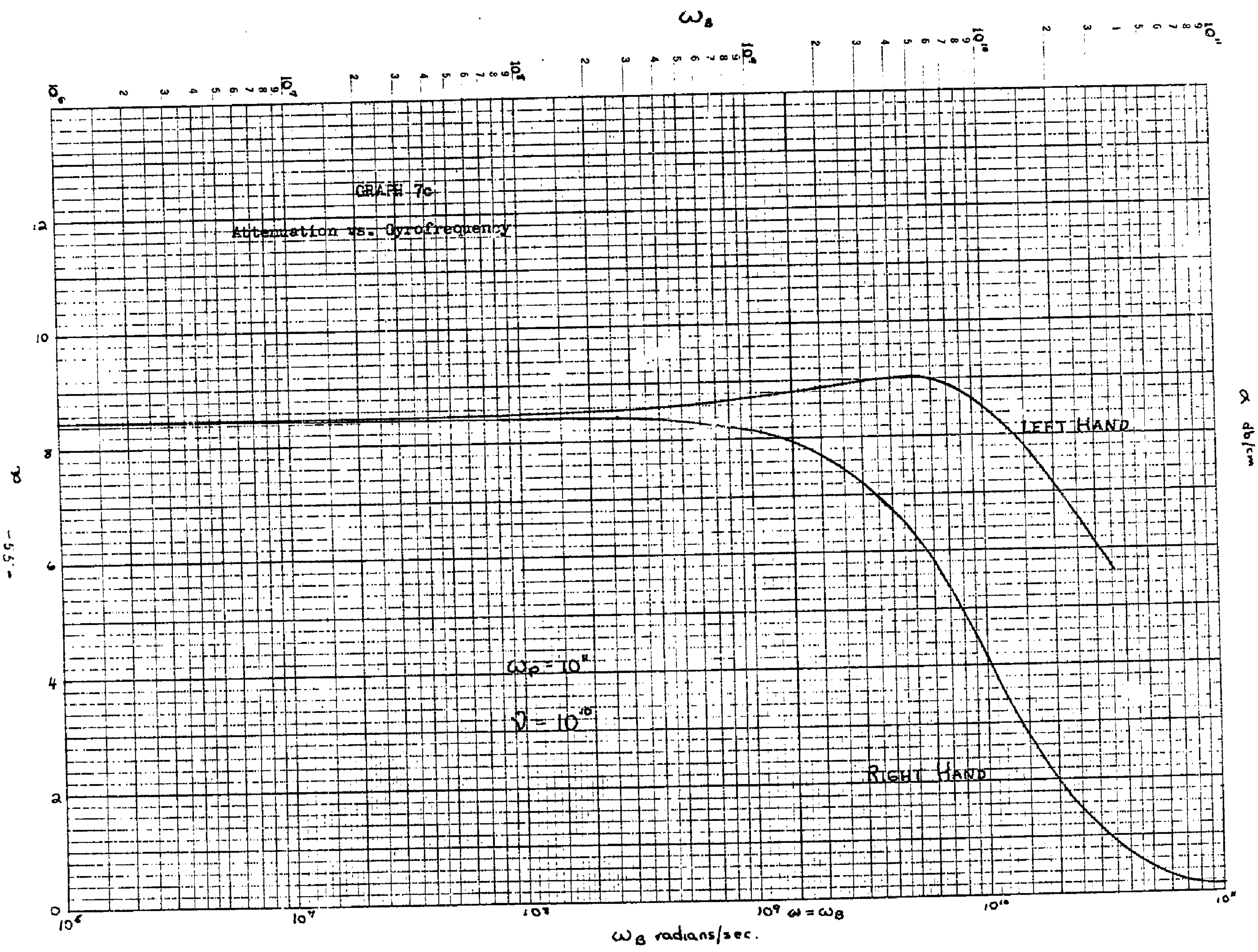

Attenuation vs. Plasma Frequency
No Magnetic Field
α
α db/cm.
ω_p radians/sec.
ω_p
$\omega_p = \omega$
$\nu = 10^{11}$
$\nu = 10^{10}$
$\nu = 10^{9}$
$\nu = 10^{7}$
$\nu = 10^{12}$
$\nu = 10^{12}$
$\nu = 10^{11}$
$\nu = 10^{10}$
$\nu = 10^{9}$
$\nu = 10^{7}$



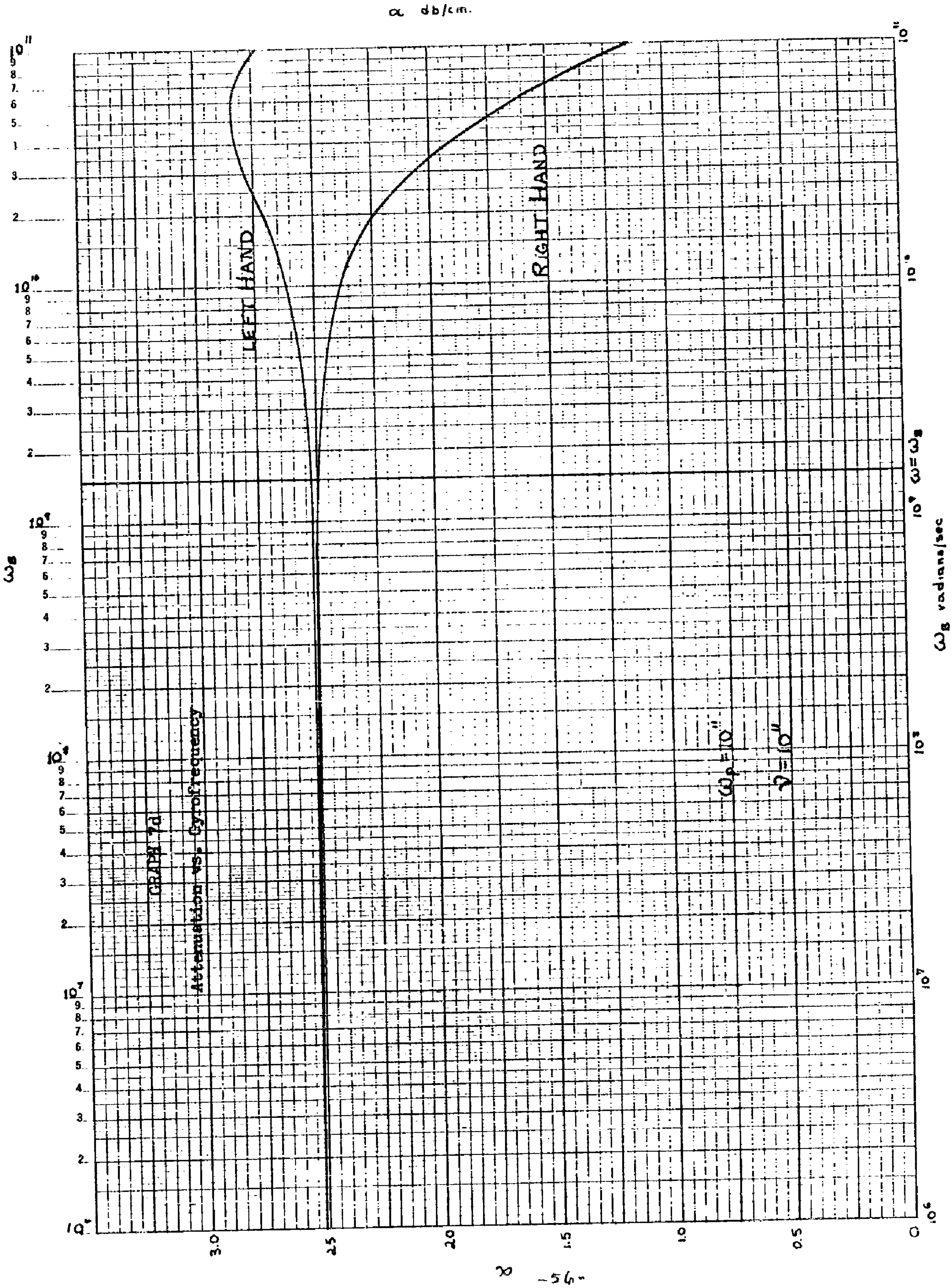
GRAPH 6
Attenuation vs. Plasma Frequency
No Magnetic Field
ω 1.507·10⁹
ν=10¹²
ν=10¹¹
ν=10¹⁰
ν=10⁹
ν=10⁷
α
α db/cm
ω_p
ω_p radians/sec


GRAPH 7a
Attenuation vs. Gyrofrequency
RIGHT HAND POLARIZATION
LEFT HAND POLARIZATION
$\omega_p = 10^{11}$
$\nu = 10^{8}$
ω_B
ω_B radians/sec.
α db/cm.
α
$\omega = \omega_B$

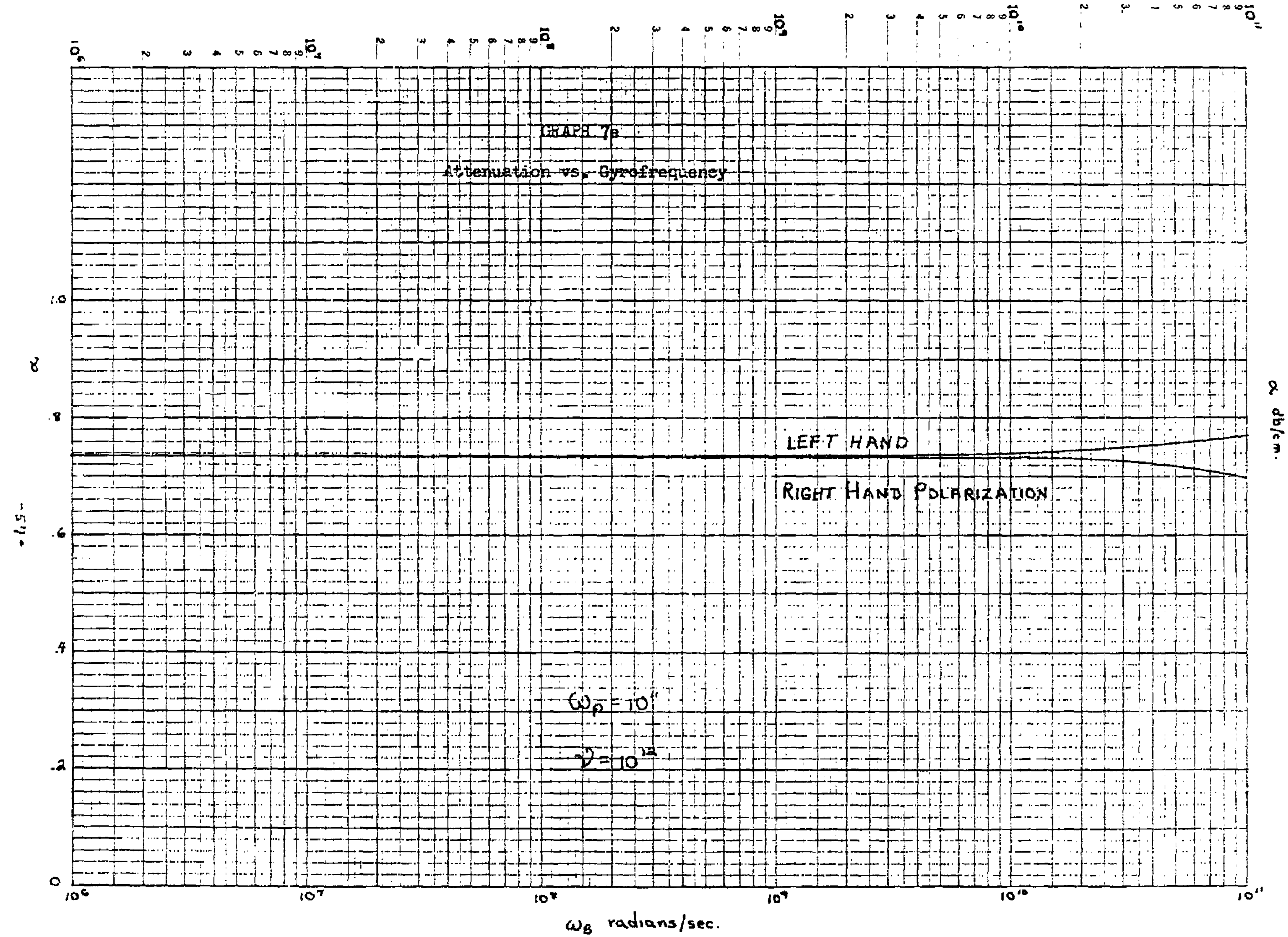

GRAPH 7b
Attenuation vs. Gyrofrequency
RIGHT HAND POLARIZATION
LEFT HAND
$\omega_p = 10^{11}$
$\nu = 10^{9}$
$\omega = \omega_B$
ω_B radians/sec.
α db/cm
ω_B



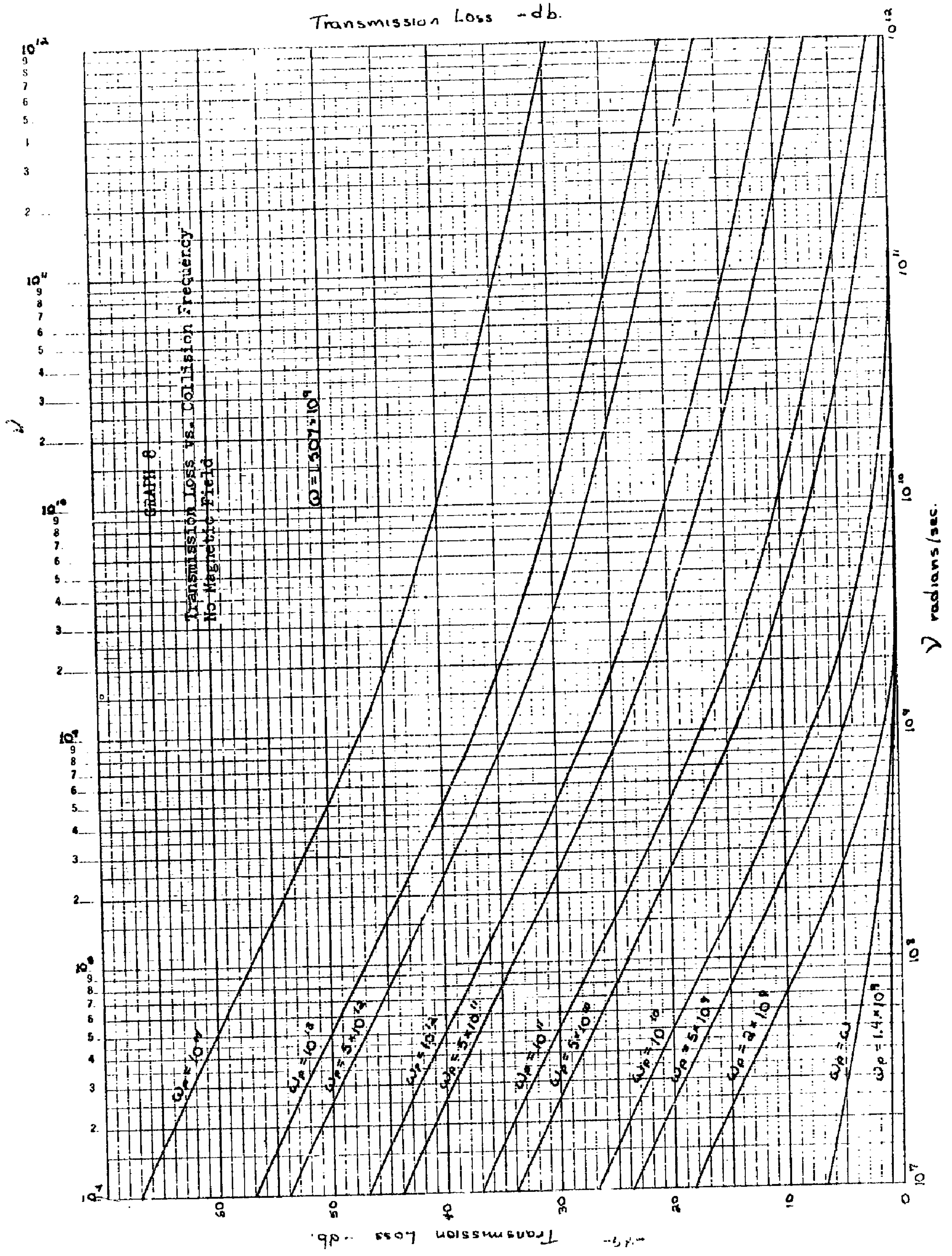
GRAPH 7c
Attenuation vs. Gyrofrequency
ω_B
α db/cm
α
LEFT HAND
RIGHT HAND
$\omega_p = 10^{11}$
$\nu = 10^{10}$
10^9 $\omega = \omega_B$
ω_B radians/sec.



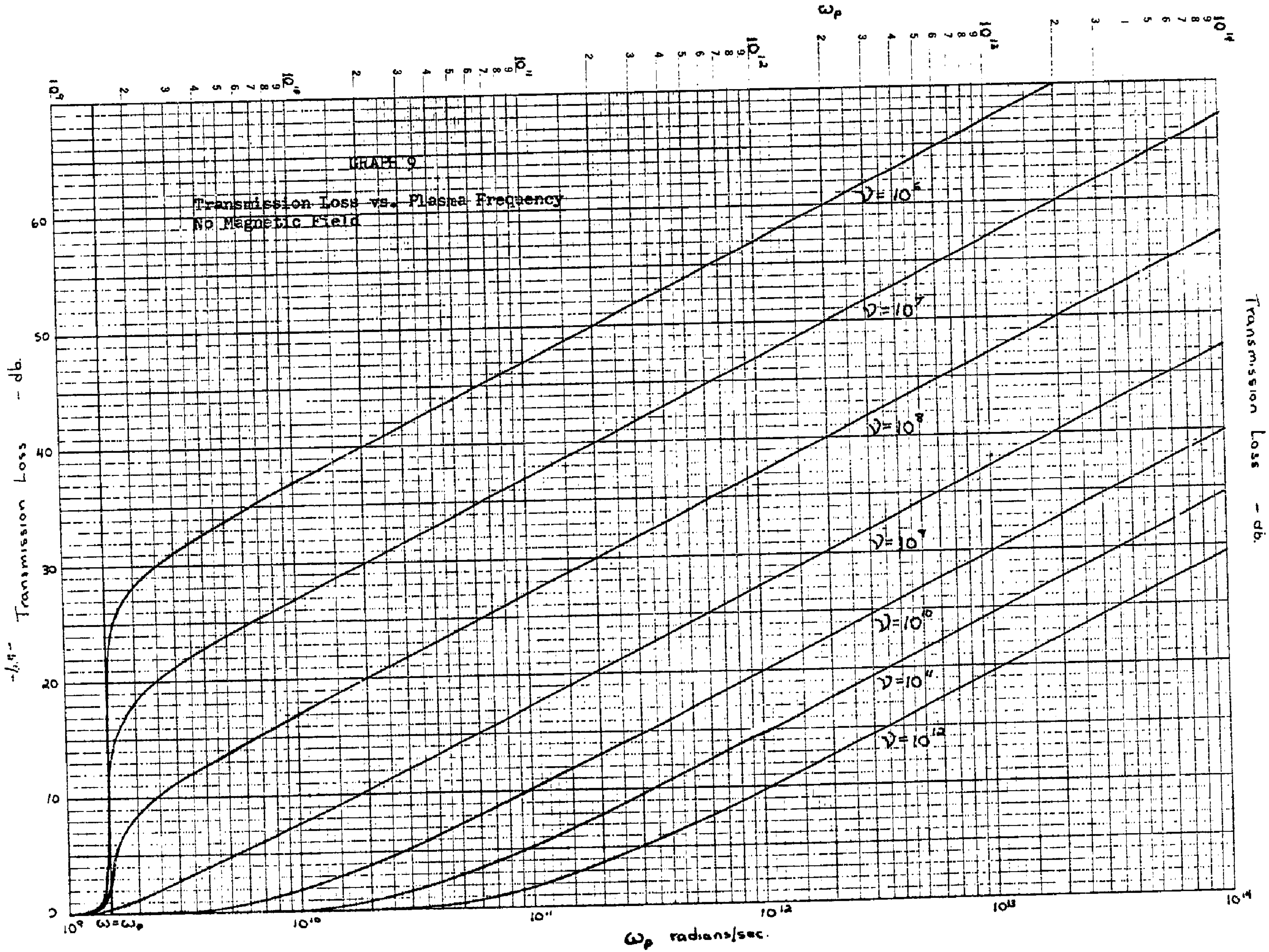
GRAPH 7d
Attenuation vs. Gyrofrequency
α db/cm.
LEFT HAND
RIGHT HAND
$\omega_p = 10^{11}$
$\nu = 10^{11}$
$\omega = \omega_B$
ω_B radians/sec
ω_B
3.0
2.5
2.0
1.5
1.0
0.5
0
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}

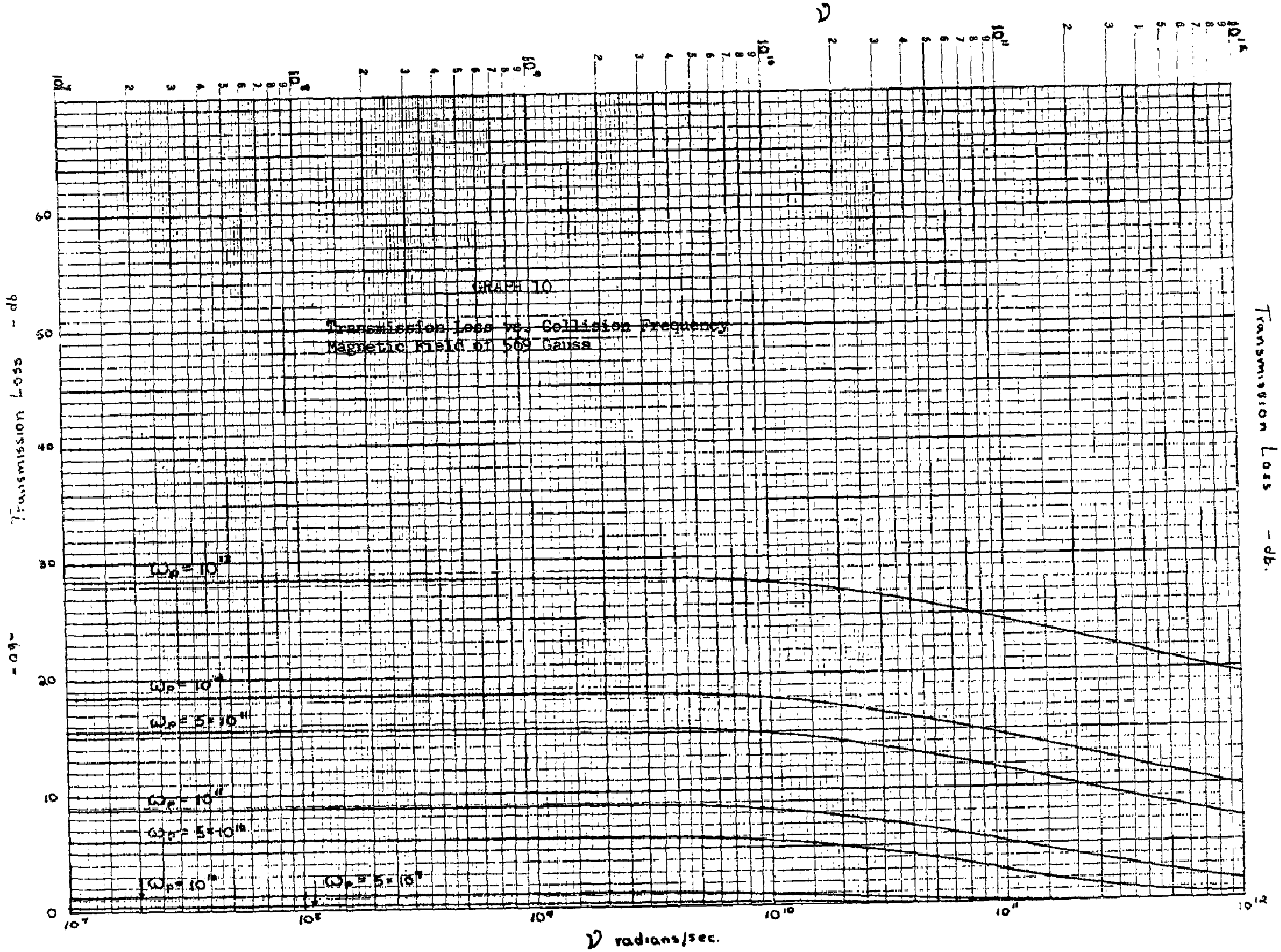


GRAPH 7B
Attenuation vs. Gyrofrequency
LEFT HAND
RIGHT HAND POLARIZATION
$\omega_p = 10^{11}$
$\nu = 10^{12}$
α db/cm.
α
ω_B radians/sec.



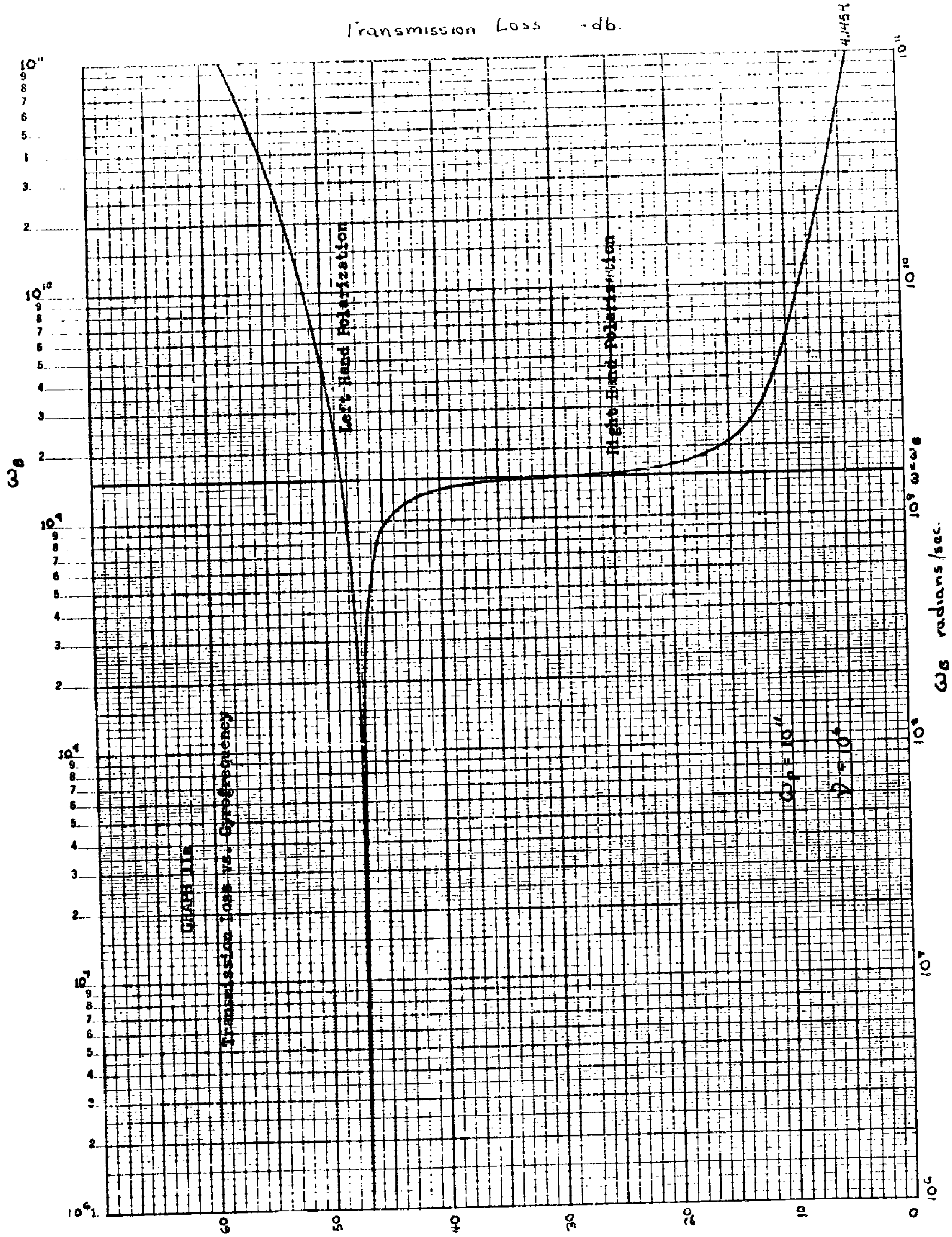
Transmission Loss -db.
GRAPH 8
Transmission Loss vs. Collision Frequency
No Magnetic Field
ω = 1.507×10^9
ν radians/sec.
ωp = 10^14
ωp = 10^13
ωp = 5×10^12
ωp = 10^12
ωp = 5×10^11
ωp = 10^11
ωp = 5×10^10
ωp = 10^10
ωp = 5×10^9
ωp = 2×10^9
ωp = ω
ωp = 1.4×10^9
60
50
40
30
20
10
0
Transmission Loss -db.


GRAPH 9
Transmission Loss vs. Plasma Frequency
No Magnetic Field
$\nu = 10^6$
$\nu = 10^7$
$\nu = 10^8$
$\nu = 10^9$
$\nu = 10^{10}$
$\nu = 10^{11}$
$\nu = 10^{12}$
Transmission Loss - db.
ω_p radians/sec.
$\omega = \omega_p$
ω_p

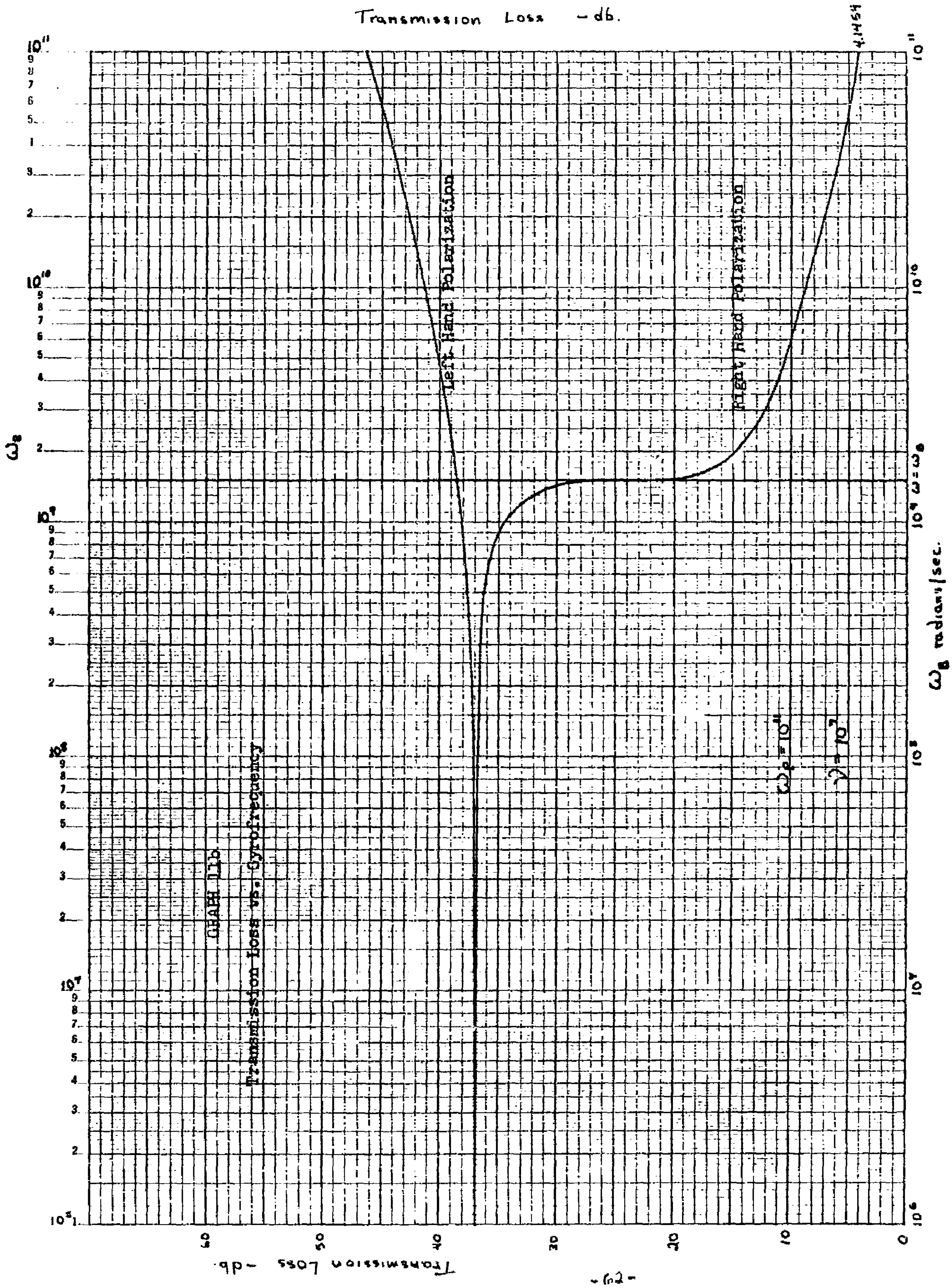


GRAPH 10

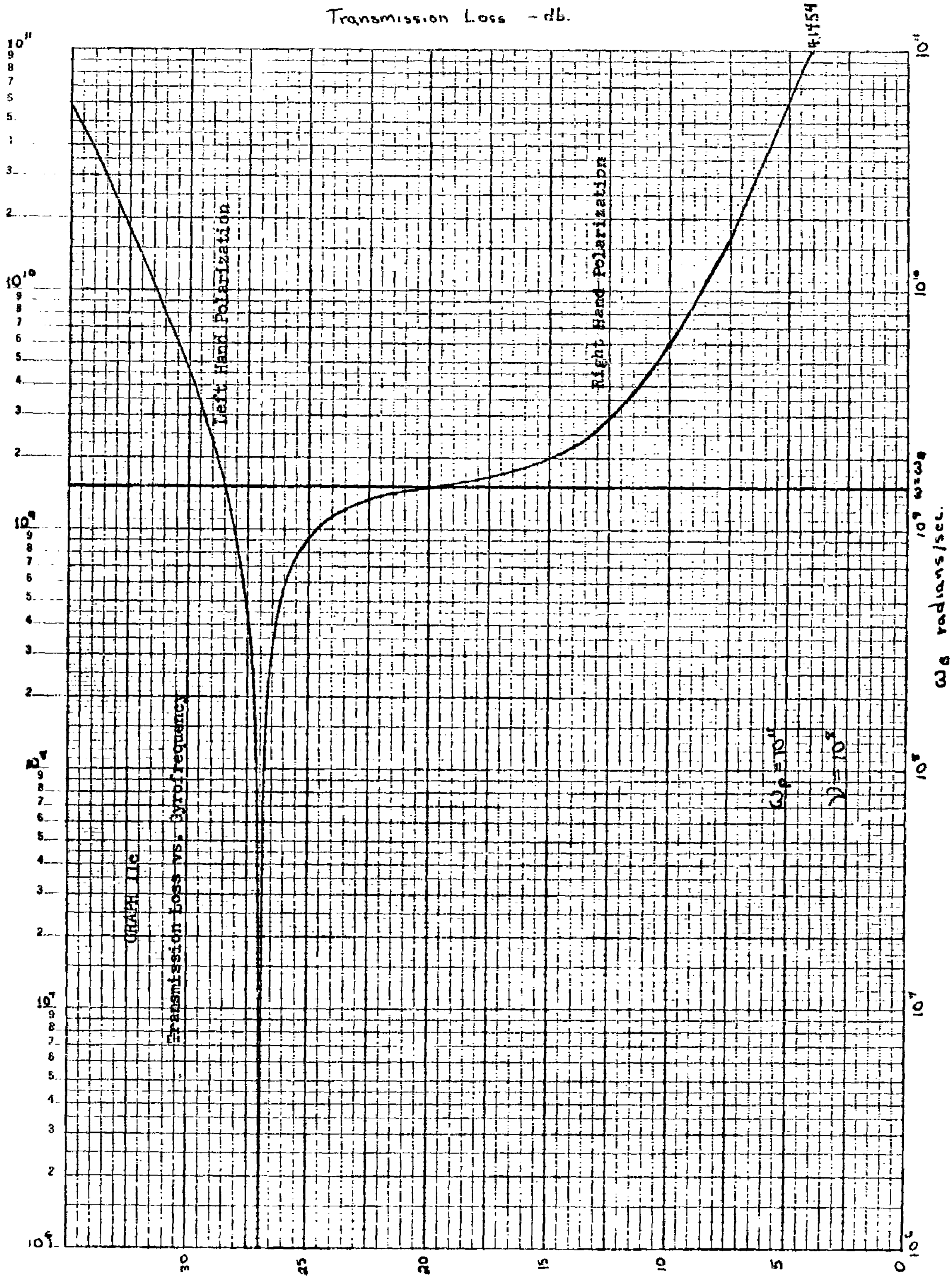
Transmission Loss vs. Collision Frequency
Magnetic Field of 509 Gauss

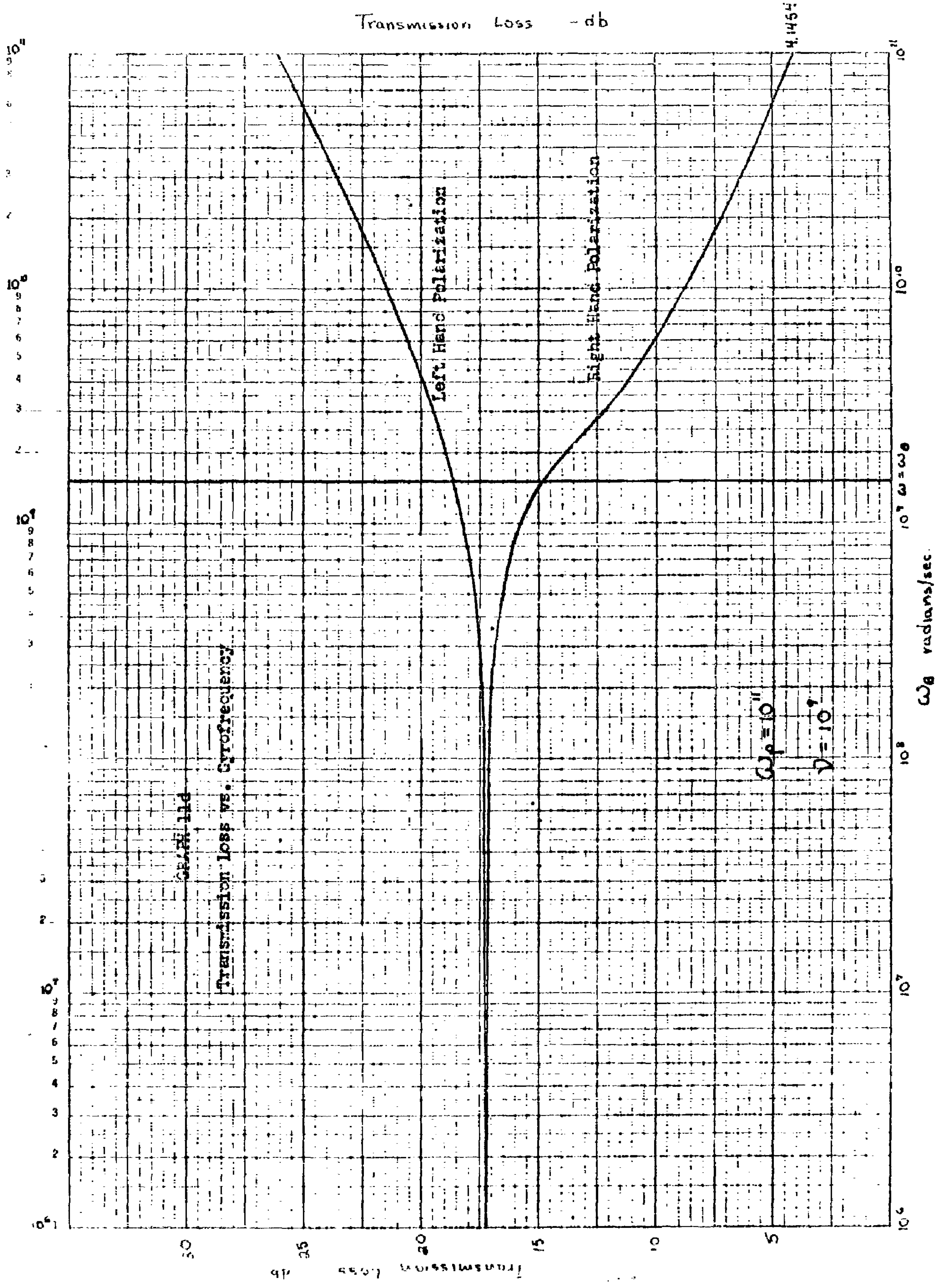


Transmission Loss -db
4.1457
Left-Hand Polarization
Right Hand Polarization
ω_B
ω_B radians/sec.
10^9 $\omega = \omega_B$
GRAPH III B
Transmission Loss vs. Gyrofrequency
$\omega_p = 10^{11}$
$\nu = 10^6$

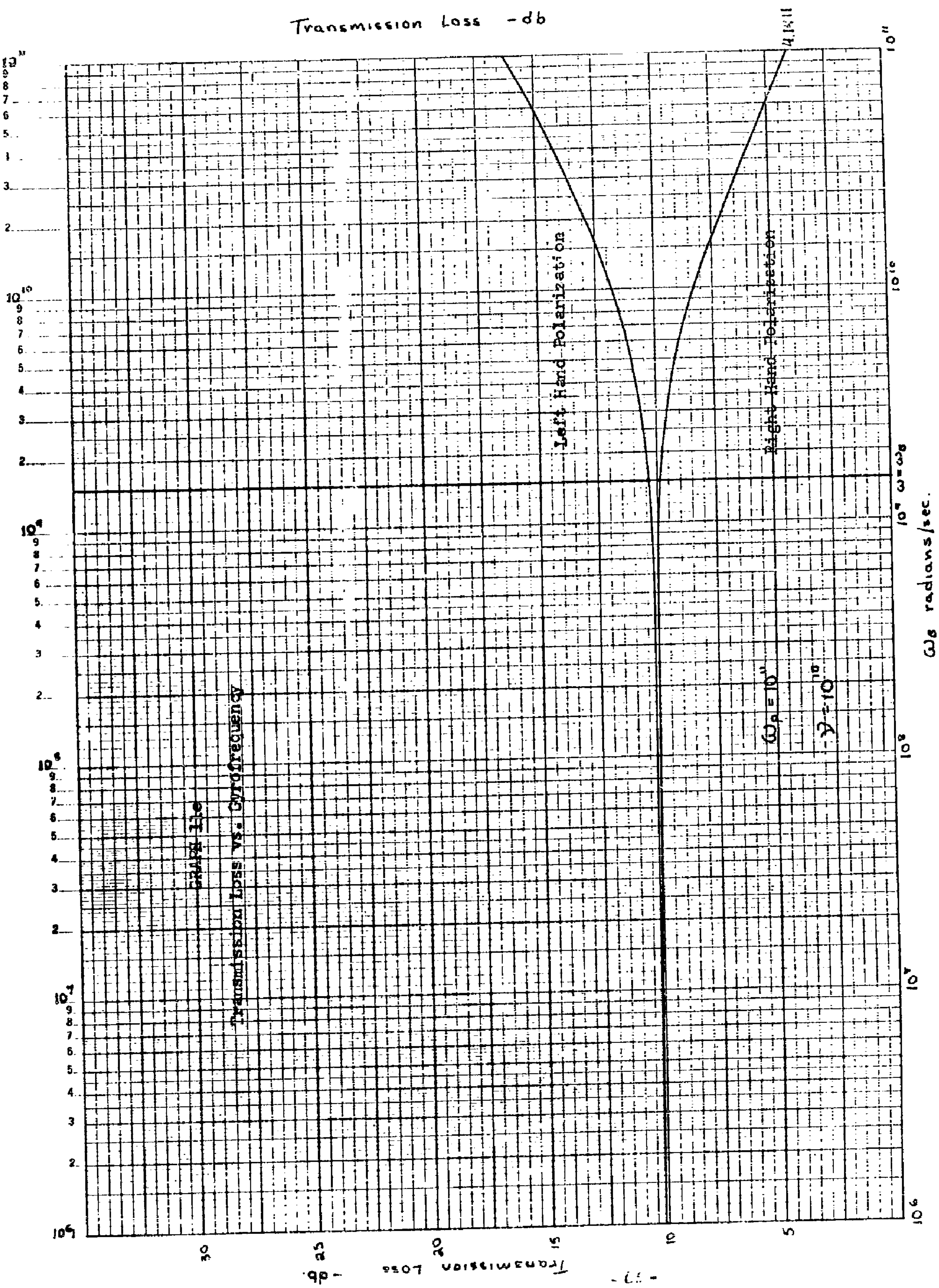


Transmission Loss - db.
GRAPH 11b
Transmission Loss vs. Gyrofrequency
Left Hand Polarization
Right Hand Polarization
ω_B
ω_B radians/sec.
$\omega = \omega_B$
$\omega_p = 10^{11}$
$\nu = 10^7$
4.1454

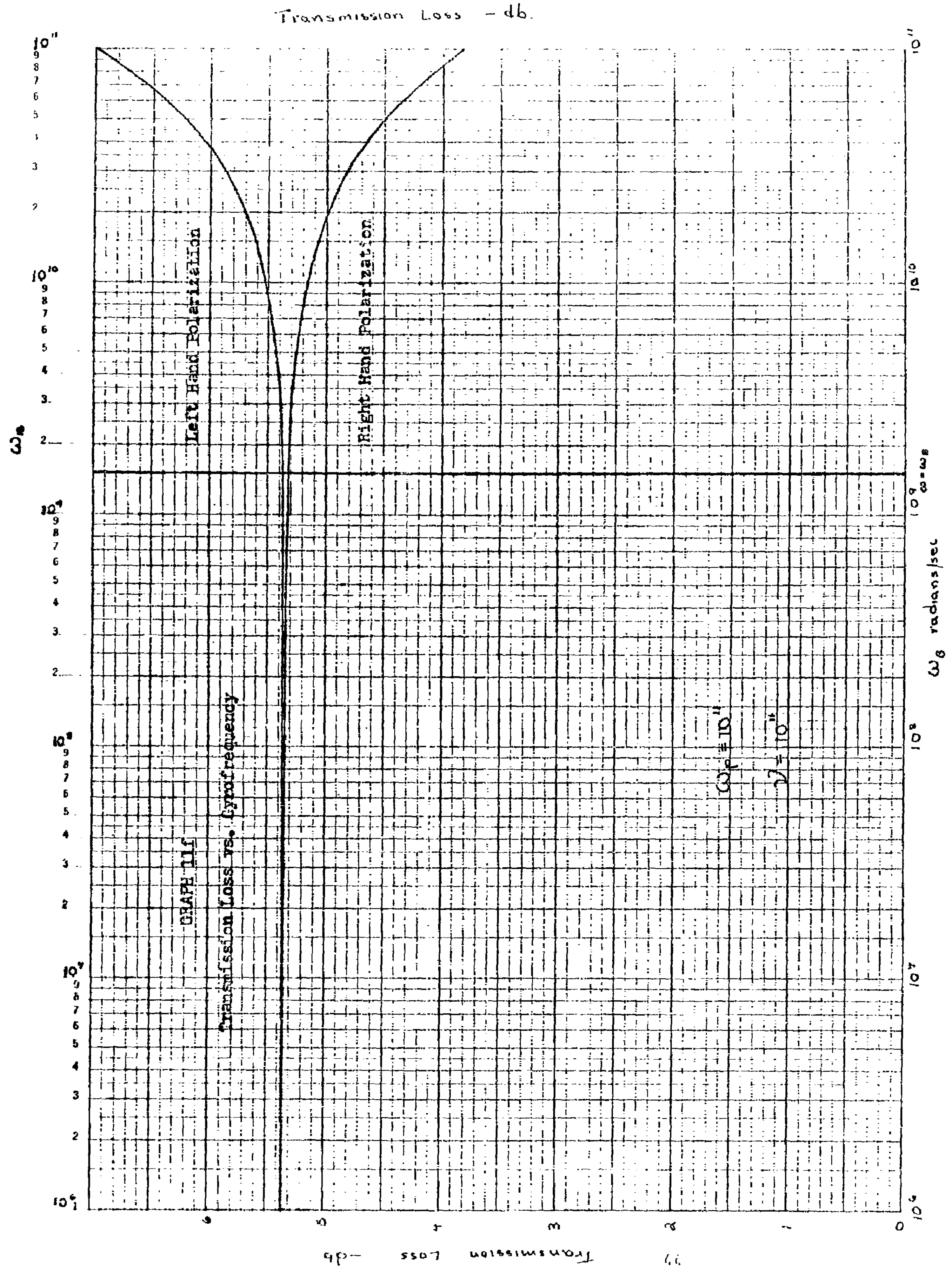

Transmission Loss -db.
4.1454
Left Hand Polarization
Right Hand Polarization
GRAPH IIc
Transmission Loss vs. Gyrofrequency
$\omega_p = 10^{11}$
$\nu = 10^8$
ω_g radians/sec.
30
25
20
15
10
5
0



Transmission Loss -db
GRAPH IId
Transmission loss vs. Gyrofrequency
Left Hand Polarization
Right Hand Polarization
$\omega_p = 10^{11}$
$\nu = 10^{9}$
ω_B radians/sec.
$\omega = \omega_B$
4.1454
30
25
20
15
10
5



Transmission Loss -db
GRAPH IIe
Transmission Loss vs. Gyrofrequency
Left Hand Polarization
Right Hand Polarization
$\omega_o = 10^{11}$
$\nu = 10^{10}$
$\omega = \omega_B$
ω_B radians/sec.
Transmission Loss -db.



Transmission Loss -db.
Left Hand Polarization
Right Hand Polarization
GRAPH 11f
Transmission Loss vs. Gyrofrequency
ω_B radians/sec
$\omega = \omega_B$

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An analysis is presented on the interaction of an electro-magnetic wave with a plasma. General equations are derived for the dielectric constant, propagation function, and conductivity of a plasma. The attenuation and reflection characteristics of a homogeneous, semi-infinite, uniform plasma slab are also considered. The analysis includes the effect of a magnetic field on attenuation and reflection.

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6. The Homogeneous Semi-Infinite Plasma Slab
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